Real-time centroid moment tensor determination for large earthquakes from local and regional displacement records

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SUMMARY
We present an algorithm to rapidly determine the moment tensor and centroid location for large earthquakes employing local and regional real-time high-rate displacement records from GPS. The algorithm extracts the coseismic offset from the displacement waveforms and uses the information to invert for the moment tensor. The Green’s functions for a layered earth are obtained numerically from open source code EDGRN. To determine the centroid, multiple inversions are simultaneously performed within a grid of inversion nodes, and the node with the smallest misfit is then assigned the centroid location. We show results for two large earthquakes replayed in simulated real-time mode using recorded 1 Hz GPS displacements: the 2003 $M_w 8.3$ Tokachi-oki and the 2010 $M_w 7.2$ El Mayor-Cucapah earthquakes. We demonstrate that it is feasible to obtain accurate CMT solutions within the first 2–3 min after rupture initiation without any prior assumptions on fault characteristics, demonstrating an order of magnitude improvement in latency compared to existing seismic methods for the two earthquakes studied. This methodology is useful for rapid earthquake response, tsunami prediction and as a starting point for rapid finite fault modelling.

Key words: Inverse theory; Satellite geodesy; Earthquake source observations; Early warning.

1. INTRODUCTION
Computation of the seismic moment tensor (MT) for a given earthquake is one of the fundamental kinds of modelling that can be performed. The MT can be calculated from a number of methods such as polarity of first arrivals (Havskov & Ottemoller 2010) or waveform matching (Dreger 2003) and is a compact representation of the earthquake source that contains basic information on the size of the event, the fault plane geometry and the style of faulting. MT solutions are of use over a range of earthquake magnitudes. Small-to-medium events are utilized for tectonic studies and to determine the stress regime within a region. For large events, rapid determination of the centroid location as well as the moment tensor (CMT) provides valuable information for earthquake response, tsunami early warning and as a starting point for finite fault source modelling.

Currently there are a number of efforts that routinely compute MT solutions for earthquakes worldwide. The most comprehensive catalogue of such solutions is contained in the Global Centroid Moment Tensor (GCMT) Project. At its inception the GCMT project included inversion of body and surface waves (Dziewonski et al. 1981; Dziewonski & Woodhouse 1983) and has seen numerous refinements since, such as inclusion of apherical earth structure, attenuation, etc. This method however employs only teleseismic data and its emphasis is in data collection and catalogue compilation not in rapid modelling. Real-time MTs can be obtained for small-to-medium events using time domain waveform matching inversion schemes (Dreger & Helmberger 1990; Dreger 2003). However, real-time CMT determination of medium-to-large events is still an active area of research.

One of the most important advances in computing CMTs as quickly as possible for large events is contained in the work of Kanamori & Rivera (2008) who elaborated on Kanamori’s (1993) observation of the $W$ phase, a long-period phase arriving in between the direct $P$ and $S$ waves. They showed that inversion for the MT using data as close as 15° from the source is viable. $W$-phase inversion algorithms currently run in real time at the USGS, Pacific Tsunami Warning Center (PTWC) and Institut du Physique du Globe de Strasbourg (IPGP-EOST, Hayes et al. 2009). Since the $W$ phase arrives well before large amplitude surface waves and remains on-scale far longer such inversion algorithms have shown to be a marked improvement in rapid computation of MT solutions for large events over traditional waveform matching techniques. Following the $M_w 9.0$ Tohoku-oki earthquake, Duputel et al. (2011) showed that it was feasible to use data at distances as small as 11° from the source and there are indications that, had regional data been available in real time, broad-band recordings as close as 7° might have been useful as well (H. Kanamori private communication, 2011). However, $W$-phase-based inversion schemes, while very robust, require long-period displacement records (e.g. 200–1000 s for the 2011 Tohoku-oki event; Duputel et al. 2011), these are almost always unusable close to the source in real time for...
well-known reasons; velocity instruments clip and it is difficult to extract long-period motions from strong-motion accelerometer data because of tilts and rotations of the instruments (Boore & Bommern 2005).

Thus, there seems to be a limitation in how fast MT solutions can be obtained operationally for large events using seismic instruments and existing seismological methods. For example, for the Tohoku-oki event it took 20 min after origin time to arrive at the first CMT solutions by agencies running W-phase algorithms (Duputel et al. 2011), even though the rupture had a duration of 2 min (Simons et al. 2011). This delay was due to the reliance on teleseismic data. After several iterations using progressively more data, the final CMT solution was obtained by the National Earthquake Information Centre (NEIC) 90 min after origin time using data up to 90° from the rupture. The first estimate of moment magnitude was obtained in about 3 min by the Japan Meteorological Agency (JMA), but was grossly underestimated at $M_w = 8.0$. Duputel et al. (2011) documented that in the numerous iterations between agencies the nodal planes were somewhat consistent with only minor variations in strike dip and rake, the magnitudes oscillated between $M_w = 8.8$ and 9.0 after the 20-minute mark, but the centroid locations varied by as much as 2° and 60 km in depth.

Permanent deformation is essentially a zero-frequency wave and thus the best long-period source of information about an earthquake. GPS is well equipped to characterize an earthquake as first demonstrated for magnitude estimation of the 1992 $M_c 7.3$ Landers earthquake in southern California (Blewitt et al. 1993; Bock et al. 1993). There have been some attempts to address the use of GPS static deformation estimates into rapid source modeling, notably by Blewitt et al. (2006) who showed that given an epicentral location and assuming thrust faulting for the 2004 $M_c 9.2$ Sumatra–Andaman earthquake, one could have estimated an accurate magnitude within 15 min of the origin time using global GPS stations at regional to teleseismic distances.

We present a fast and robust method for determining CMT solutions, based on real-time high-rate displacement data from near-source GPS stations. Although we are not explicitly solving for the style and geometry of faulting, that information is implicit in the MT solution. In general, we do not require prior knowledge of the sense or extent of faulting, although that information could be used if available.

We demonstrate the new algorithm by replaying the estimation of 1 Hz displacements for the 2003 $M_w 8.3$ Tokachi-oki earthquake using GPS data from Japan’s GPS network (GEONET, Miyazaki et al. 1998) and for the 2010 $M_w 7.2$ El Mayor-Cucapah earthquake using data from the California Real Time Network (CRTN, Bock et al. 2011). GPS data for the 2011 $M_w 9.0$ Tohoku-oki earthquake at the time of this study were not yet in the public domain, but our algorithm can be easily extended to this great event.

## 2. CENTROID MT INVERSION FROM DISPLACEMENT RECORDS

### 2.1 GPS waveforms

High-rate (1 Hz or greater) GPS displacement time-series are becoming more prevalent in seismology; however their use in real-time seismological applications is still very limited. We analysed in a simulated real-time mode 1 Hz GPS RINEX files for the Tokachi-oki and El Mayor-Cucapah events using the method of instantaneous positioning (Bock et al. 2000). This approach uses doubly differenced dual-frequency GPS phase information and resolves dual-frequency integer-cycle phase ambiguities on an epoch-by-epoch basis within the limitations described by Bock et al. (2011). The GEONET and CRTN networks were divided into more manageable subnetworks and then combined and referenced to a far away, stable station through a network adjustment (Crowell et al. 2009). For the Tokachi-oki earthquake, we used data from 356 stations within GEONET; all located within Hokkaido and Honshu islands. The reference station chosen was 0247 (36.8654°N, 138.1987°E), located 760 km southwest of the hypocentre, just north of Nagano on central Honshu Island. For the 2010 El Mayor-Cucapah earthquake, we used 105 CRTN stations with station GNPS (34.3086°N, 114.1895°W) near Lake Havasu as a reference site as described in Bock et al. (2011).

Our inversion scheme uses coseismic offsets so we apply a 120 s moving average filter to the 1 Hz displacement waveforms to remove the dynamic component and retain the information on the permanent deformation. GPS data have much higher noise levels than traditional seismological data sets in particular in the vertical direction, thus, we set a threshold of 15 mm on the total horizontal component at a given station. This is roughly three times the usual noise level in the horizontal component of real-time GPS measurements (e.g. Genrich & Bock 2006). At any given epoch only stations over this threshold are considered for the inversion.

### 2.2 Inversion scheme

The inversion scheme employed here relates the coseismic offset measured at the surface to source parameters at depth. Amoroso et al. (2004) and Hearn & Bürngmann (2005) showed that crustal layering can have a significant effect when inverting for source parameters using static offsets, thus we must account for, at least, a simple 1-D structure. To do so we compute Green’s functions (GFs) using Fortran codes EDGRN/EDCMP (Wang et al. 2003). This numerical approach starts from the closed form solutions of the partial differential equations of motion obtained from the Hankel transform and then applies a Thomson–Haskell propagator matrix to relate the deformation at depth with that at the surface. We extract the GFs from the code output (see the Appendix for a detailed explanation on how to harvest this information) and set up the kernel matrix $G$ for the inversion

$$
\begin{bmatrix}
\mathbf{u}_1^1 \\
\mathbf{u}_2^2 \\
\vdots \\
\mathbf{u}_5^5 \\
\mathbf{u}_1^2 \\
\mathbf{u}_2^3 \\
\vdots \\
\mathbf{u}_5^4 \\
\end{bmatrix} = 
\begin{bmatrix}
G_{11}^1 & G_{12}^1 & G_{13}^1 & G_{14}^1 & G_{15}^1 \\
G_{21}^2 & G_{22}^2 & G_{23}^2 & G_{24}^2 & G_{25}^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
G_{51}^5 & G_{52}^5 & G_{53}^5 & G_{54}^5 & G_{55}^5 \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{m}_1 \\
\mathbf{m}_2 \\
\vdots \\
\mathbf{m}_5 \\
\end{bmatrix},
\end{array}
$$

or more succinctly

$$
u_i^j = G_{ij}^k m_j; \{i = x, y, z; j = 1, 2, \ldots, 5; k = 1, 2, \ldots, n\}.
$$

where $u_i^j$ is the $i$th component of displacement measured at the $k$th station, $m_j$ is the $j$th component of the moment tensor and $G_{ij}^k$ are the $i$th component GFs that relate the $j$th component of the MT to the $k$th station. Thus the GF matrix is very compact, having only five elements per direction of motion per station.

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The MT in this case is composed of five components since we restrict the inversion to the deviatoric portion such that for the general six component Cartesian MT

\[
M = \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{xy} & m_{yy} & m_{yz} \\ m_{xz} & m_{yz} & m_{zz} \end{bmatrix},
\]  

the deviatoric restriction means that the following equivalences between eqs (2) and (3) hold

\[
m_1 = m_{xy},
\]

\[
m_2 = m_{xz},
\]

\[
m_3 = m_{yz},
\]

\[
m_4 = 0.5 \cdot (m_{xz} - m_{yz}),
\]

\[
m_5 = m_{yz}.
\]

We retain the Aki & Richards (2002) convention that x is north, y is east and z is down. This is just one of many possible MT coordinate representations; however we have chosen this one to be consistent with the notation used by the EDGRN/EDCMP software.

Since we are making a point source approximation and neglecting fault finiteness, we assume that despite the coseismic motions the source to receiver distances remain unchanged and so the GF matrix remains unaltered throughout the inversion process. Next we assemble the data vector from that epoch’s measured coseismic offset and weigh the data by the pre-event standard deviations as

\[
\text{Wu} = W \text{Gm},
\]

where

\[
W = \text{diag} \left( \frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \frac{1}{\sigma_3}, \frac{1}{\sigma_4}, \frac{1}{\sigma_5}, \ldots, \frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \frac{1}{\sigma_3} \right),
\]

and the \( \sigma_i \)'s are the standard deviations obtained from 60 s of pre-event noise at the \( k \)th station on the \( i \)th channel. This is a reasonable assumption since, in the absence of motion, the pre-event time-series are many realizations of a zero measurement. The weight matrix remains constant across all epochs, since we assume that the noise characteristics of the GPS time-series are the same for the duration of the inversion. The noise characteristics of real-time GPS displacements should be stable on the scale of minutes (Genrich & Bock 2006). Furthermore, Bock et al. (2011) using shake table testing found no appreciable increase in the noise level between quiescent periods and periods of shaking. Thus, to first order, the noise can be assumed to be constant for the duration of strong shaking.

An additional weighting is applied based on the distance \( r \) from the source to the receiver. Because the static field decays according to \( 1/r^2 \) (Aki & Richards 2002) we divide each time-series by a weight \( w_0 \) to avoid having the largest offsets overwhelm the norm minimized by the inversion. This is a technique analogous to the one used in time domain waveform MT inversion (Dreger 2003). For a centroid to station distance \( r_i \), the weight is defined as

\[
w_i^2 = \frac{\min(r_i)^2}{r_i^2}.
\]

Thus, we have two weighting factors, one that determines how trustworthy an offset is when compared to background noise levels and a second one that ensures that the largest offsets do not dominate the inversion.

The inversion is performed at each time step (once per second in this case) utilizing the coseismic offset measured at that epoch to produce a new MT. We experimented with \( L_2 \)-norm inversion using a QR decomposition and \( L_1 \)-norm inversion using the L1 Magic suite of Matlab codes (Candes & Romberg 2005), which recast the problem as a linear program and then implement the primal-dual algorithm described in Boyd & Vandenberghe (2004). We found that \( L_1 \)-norm minimization converges to a stable solution before the \( L_2 \)-norm inversion, however, we have programmed both solutions and included them in the code we make available.

For analysis of the inversion we obtain the seismic moment \( M_0 \) as the scaled Frobenius norm of the MT (Silver & Jordan 1982)

\[
M_0 = \frac{1}{\sqrt{2}} \left( \sum_{j=1}^{3} \sum_{i=1}^{3} M_{ij}^2 \right)^{1/2},
\]

and included them in the code we make available.

\[
\text{VR} = \left( 1 - \frac{\sum_{i=1}^{n} (d_i^2 - (\text{Gm}))^2}{\sum_{i=1}^{n} d_i^2} \right) \times 100.
\]

To build the grid of nodes on which the inversion will take place, one can use a pre-computed slab model (in the case of subduction zone events) or have a library of fault surfaces (for strike-slip environments) as a template for a grid. One could then discretize the known geological surfaces to define the inversion nodes thus forcing the centroid to lie on known faults. Nonetheless it must be noted that employing regional tectonics as a guide to form the inversion grid is not strictly necessary. Alternatively, as we demonstrate in this paper, for subduction and strike-slip events one can simply build a sufficiently large 3-D prism of gridpoints around a preliminary hypocentral location. The choice will depend on the observational goals of a network. In any case, in this paper we combine a formal inversion with a grid search to solve for the CMT using an algorithm that we call fastCMT.
3. RESULTS

To demonstrate our approach, we apply here the fastCMT algorithm to a subduction zone earthquake and to an earthquake in a strike-slip environment, using near-field 1 Hz GPS network data replayed in a simulated real-time mode to estimate displacements.

3.1 2003 $M_w$ 8.3 Tokachi-oki earthquake

Our first example of fastCMT is for the 2003 $M_w$ 8.3 Tokachi-oki earthquake. This megathrust event ruptured a segment of the Kuril–Japan trench, sharing most of the source area and rupture characteristics of the 1953 $M_w$ 8.1 Tokachi-oki earthquake (Hamada & Suzuki 2004). We estimated displacements in a simulated real-time mode for 300 s of GEONET 1 Hz data from 355 stations on Honshu and Hokkaido islands. Some very near source stations lost telemetry and have incomplete records so we excluded those from processing. We applied a 120 s moving average to each displacement record in each coordinate component to extract the permanent deformation from the displacement waveforms; the resulting time-series can be seen in Fig. 1. The static offset at the stations closest to the source is discernible at around 160 s. As is usual with GPS time-series, the vertical component is noisier than the horizontals (e.g. Genrich & Bock 2006).

GFs were computed from EDGRN on a 1 km horizontal and vertical grid using the four-layer velocity model employed by Yagi (2004) for near source slip inversion (Table 1). This is a much denser coverage than the actual station distribution, thus, at any given station the resulting GF is the spline interpolation of the closest gridpoints. For this scenario we used a 3-D $3^\circ \times 3^\circ$ prism of gridpoints spaced at 0.2$^\circ$ and every 3 km in depth between 3 and 60 km centred around the mean latitude and longitude of the first stations to meet a detection criterion.

The criterion that we applied is that displacements from five stations over the 15 mm threshold are required to start the inversion; for the 2003 Tokachi-oki event this occurred at 43 s after origin time. The results of the inversion are shown in Figs 2 and 3 and Movie S1. Fig. 2 shows the centroid determination as a function of time as well as the estimated magnitude and the parameter $\epsilon$; Fig. 3 shows snapshots of the resulting CMT (the full result is in Movie S1) as well as the observed and synthetic horizontal displacements. Fig. 2 shows that by 50 s a rough centroid location is available with oscillations between adjacent nodes. The magnitude reaches $M_w$ 8.0 at 75 s, with a 75 per cent VR. However, as evidenced by the time-series (Fig. 1) the full coseismic offset has not yet occurred, most likely because of the effects of the moving average filter. The magnitude continues to grow and reaches at 8.3 by 170 s and slowly declines to 8.2 for the remainder of the inversion. The plot of the VR also shows that at 200 s (when the final offset is in place) the fit to the data is maximum ($\sim$85 per cent) degrading towards the end of the inversion. Fig. 3 and Movie S1 show the oscillation between adjacent nodes for the centroid solution, and also indicate how by 65 s a thrusting mechanism is already resolved although the magnitude is still underestimated. However, by 180 s and onwards the inverted mechanism is close to that of the GCMT.

### Table 1. Velocity model used for the Tokachi-oki inversion.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_p$ (km s$^{-1}$)</th>
<th>$V_s$ (km s$^{-1}$)</th>
<th>Density (kg m$^{-3}$)</th>
<th>Thickness (km)</th>
</tr>
</thead>
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<td>2.6</td>
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</tr>
<tr>
<td>4</td>
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<td>7.8</td>
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<td>$\infty$</td>
</tr>
</tbody>
</table>

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Real-time CMT calculation from displacement records

To further evaluate the quality of the solution we extracted the strike, dip and rake of the nodal planes from the MT solution every second by decomposing it into its best double couple. The results are shown in Fig. 4 and compared to the GCMT results. They illustrate that the geometrical parameters of the best double couple are well determined at 65 s, before the full coseismic deformation occurs, and remain fairly similar to the post-processed GCMT solution throughout. Furthermore the size of the CLVD is fairly small throughout the inversion, never exceeding $\varepsilon = 0.05$.

3.2 The 2010 $M_w$ 7.2 El Mayor-Cucapah earthquake

The 2010 $M_w$ 7.2 El Mayor-Cucapah earthquake ruptured roughly 120 km of the Pacific-North America plate boundary in northern Baja California, Mexico. The rupture was complex possibly starting with a normal faulting event followed by simultaneous normal and right lateral faulting (Hauksson et al. 2010). The rupture plane showed evidence of a warped fault, was bilateral and the rake changed along strike away from the epicentre from pure strike-slip to a mixture of strike-slip and dip-slip motion (Wei et al. 2011).

Displacements at a 1 Hz sampling rate were estimated in a simulated real-time mode for 105 stations of the CRTN in southern California as described by Bock et al. (2011). As in the first event, we applied the 120 s moving average filter, at each epoch we exclude stations with horizontal offsets smaller than 15 mm (Fig. 5), and started the inversion process when five stations detected motion over the threshold; at 41 s after origin time for the 2010 El Mayor-Cucapah event. The horizontal coseismic offsets are apparent at the stations closer to the source by 150 s in both components although some shaking is still visible in the form of small oscillations. By 200 s the shaking ceases and only the offset remains. Offsets are not easily discernible in the vertical components, which remain noisy throughout. This is reasonable since this earthquake produced only small vertical offsets of $\approx$ 5 mm from the stations closer to the source as estimated, for example, from the more accurate, standard 24-hr displacement time-series computed by JPL and SIO and accessible on the GPS Explorer data portal (http://geoapp03.ucsd.edu/gridsphere/gridsphere).

The seismic velocity model used in the inversion is obtained from a simplification of the California Community Velocity Model version 4 (CVM4) (Kohler et al. 2003). The structure in this zone is highly heterogeneous so our model is an average of the grid with corners (117° W, 32° N) and (115° W, 34° N). We assume a constant Moho depth, which is taken as the mean value inside the grid. We define four layers between the Moho and the free surface, and assume a half-space below the Moho (Table 2). The GFs for this model are computed at 1 km horizontal and vertical intervals with EDGRN and as before, at a particular station the GF is the result of the spline interpolation of the closest gridpoints.

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Figure 3. Snapshots of the inversion results comparing the Global CMT result with the replayed GPS real-time inversion for the 2003 $M_w$ 8.3 Tokachi-oki earthquake. $M_w$ is the moment magnitude and VR the variance reduction. The red dots indicate the nodes of the $3^\circ \times 3^\circ$ grid used for the grid search. Also shown is the comparison between the observed (blue) and modelled (green) horizontal offsets.

To avoid the use of a library of fault surfaces we place a rectangular $3^\circ \times 3^\circ$ grid of inversion nodes around the mean latitude and longitude of the first five stations to detect 15 mm of horizontal motion and spaced at 0.1$^\circ$ in latitude and longitude and 2 km in depth from 2 to 20 km. As before, we prefer an $L_1$-norm inversion. Fig. 6 shows a summary of the centroid determination and magnitude results of the inversion. The centroid is well located by 50 s. After 50 s the centroid location oscillates between adjacent nodes; the depth oscillates between 2 and 8 km before settling at 2 km by 280 s. The VR is maximum (~85 per cent) by 150 s and worsens slightly towards the end of the inversion, the style of faulting is not well resolved until ~150 s (Figs 7 and 8, Movie S2). The moment magnitude estimate reaches 7.2 by 60 s but continues to grow and oscillates between 7.0 and 7.5, before becoming fairly stable at 7.2 by 160 s.

The focal mechanisms display some interesting characteristics during the inversion; Fig. 7 shows snapshots of the inversion and centroid location at 50 s intervals and Fig. 8 shows the
Figure 4. Geometrical parameters of the best double couple solution extracted from the MT inversion and compared to the global CMT post-processed result. Data for both nodal planes (NP1 and NP2) are plotted at 3 s intervals for clarity. $\varepsilon$ is defined in eq. (9). Data start at 43 s when enough stations detect an offset as described in Section 4.

Figure 5. 300 s of displacement records at 50 of the 105 CRTN stations that exceeded the 15 mm threshold with a 120 s moving average filter applied for the 2010 $M_w$ 7.2 El Mayor-Cucapah earthquake.
Table 2. Velocity model used for the El Mayor-Cucapah inversion.

<table>
<thead>
<tr>
<th>Layer</th>
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</table>

strike, dip and rake of the best double-couple solutions (Movie S2 shows the full result). By 120 s the best double-couple solution is almost pure strike-slip with strike and dip very similar to the $W$-phase solution and a centroid $30$ km from the $W$-phase centroid (http://earthquake.usgs.gov/). The strike of both nodal planes remains fairly consistent throughout as does the rake and the dip of the first nodal plane. However the dip of the second nodal plane oscillates consistently throughout the inversion. Furthermore, the size of the CLVD component is quite large with a mean value of 0.25 during the inversion period. This is consistent with the observations of Hauksson et al. (2010) who obtain similar results from post-mortem $W$-phase inversions and analysis of satellite geodesy data (Wei et al. 2011), which indicate that a large CLVD reflects source complexity and is a real signal.

3.3 Final rapid CMT solutions

We have shown that the final CMT solution is quickly obtained after the end of the rupture process as soon as the full permanent deformation has been discerned. The convergence to the final CMT solution is illustrated in Fig. 9 for both earthquakes. The procedure is initiated as described earlier when five stations have a total horizontal displacement that exceeds the 15 mm threshold. We refer to...
Real-time CMT calculation from displacement records

Figure 7. Snapshots of the inversion results for the El Mayor Cucapah earthquake comparing the USGS $W$-phase result with the GPS real-time inversion. Also shown is the comparison between the observed and modelled horizontal offsets. The red dots indicate the nodes used for the grid search, dark grey dots are 1 yr of aftershocks reported by the Southern California Seismic Network (www.data.scec.org) and black lines indicate Quaternary faults in southern California.

this instant as the time of first detection $t_0$; it is the time at which we launch the procedure to determine the final coseismic offsets. The mean latitude and longitude of the five GPS stations is computed, and a $3^\circ \times 3^\circ$ grid around that value is defined. The pre-computed GFs are spline interpolated to compute values at each gridpoint. A grid search is initiated to determine the event centroid. In addition to the 120 s moving average computed for the total horizontal displacements. We also compute the variance of 20 samples prior to the current epoch (Figs 9a and e); as the offset grows, the variance will be high and when the displacement stabilizes to its final level the variance will diminish and stabilize. Only the station with the maximum horizontal displacement is considered at this point of the analysis. At each epoch, the station that obeys this constraint is tracked; it can change from epoch to epoch depending on the location of the network with respect to the earthquake source. Thus, the displacement traces shown in Figs 9(a) and (e) could be a composite of several stations. The variance at each epoch is computed over the previous 20 samples (or the previous 20 s for 1 Hz data). At every instant we track whether the observed variance is the maximum observed one ($\sigma_{\text{max}}^2$) up to that given point (the time of maximum variance is $t_{\text{max}}$). Simultaneously, we track whether the variance has dropped below an empirically set threshold $p \cdot \sigma_{\text{max}}^2$, where $p = 0.25$. The epoch where that threshold is breached is time $t_1$, and we assume this to be the instant when the final offset at the station with the maximum horizontal displacement has been reached. Evidently some error is incurred here from stations not yet developing the full offset, but after experimenting with values of $p$ ranging from 0.01 to 0.5, and comparing with post-processed inversions we conclude that 0.25 is optimum. A high value for $p$ detects the final offset earlier but does not allow for enough stations to have reached a final offset, conversely a low value of 0.1 waits an unnecessarily long time for the final offset solution.

As evidenced by Figs 4 and 8, there is still a considerable amount of scatter in the MT solutions; so taking data from a single epoch is not desirable and more averaging is necessary. Thus the fastCMT algorithm waits (a somewhat arbitrary) 20 s after $t_1$. At $t_2 = t_1 + 20$, the displacements of the three directions of motion are averaged, in between $t_1$ and $t_2$ for all stations over the 15 mm threshold, and at that point the inversion procedure begins. Assuming negligible computation times, $t_2$ is also the time at which a final solution is available. Figs 9(a) and (e) show the maximum displacement and its corresponding variance function for the Tokachioki and El Mayor-Cucapah events; Figs 9(b)–(d) and 9(f)–(h) show the computed final offsets compared to the original time-series for all three directions of motion for both events. Figs 10 and 11 show the final CMT solutions for the Tokachi-oki and El Mayor-Cucapah events, respectively, and compare them to the GCMT solution in the Tokachi-oki case and

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the $W$-phase solution in the El Mayor-Cucapah case. In addition, Table 3 summarizes the times at which different milestones are reached for each event.

The origin time for the Tokachi-oki event according to the global CMT solution is 19:50:06 UTC; our fastCMT solution is available 211 s after that. The fastCMT centroid is located 4.6 km north of the GCMT, however the GCMT solution is at a depth of 28 km compared to the 48 km computed by fastCMT. From the Slab 1.0 model of Hayes & Wald (2009), the fastCMT solution, lies 10 km below the slab, while the GCMT solution lies some 10 km above the position of the slab. The moments are very similar, $2.9 \times 10^{21}$ Nm for the fastCMT solution and $3.1 \times 10^{21}$ Nm for the GCMT solution, yielding $M_w$ 8.2 and $M_w$ 8.3, respectively. The largest difference is in the geometrical characteristics of the first nodal plane. The GCMT solution has a strike of 250° while our rapid solution computes a strike of 229°. The strike in the Slab 1.0 model at 38 km depth increases smoothly from South to North from 200° at 41°N to 245° at 42.4°N, decreasing to 230° by 43°N thus the fastCMT solution seems to be closer to the strike in the slab model. Both solutions show a shallow dipping fault, 12° for fastCMT and 11° for GCMT, the rake angle is 103° for fastCMT while the GCMT solution has a significant strike-slip component with a rake of 132°. A strike-slip component seems to be supported by joint strong motion and GPS inversions (Koketsu et al. 2004), although not as large as suggested by the GCMT result. This difference is also evident in the azimuths of the principal axes although the plunge angles between the two solutions are fairly similar. Finally, it's interesting to note that the GCMT solution has a large CLVD component (10 per cent) while our rapid solution favours almost a pure double-couple solution with a CLVD of 0.4 per cent.

For the El Mayor-Cucapah earthquake we compare the fastCMT solution with the $W$-phase result available from the USGS (http://earthquake.usgs.gov/earthquakes/). The origin time for this event is 22:40:45 UTC; the fastCMT solution is available 186 s after that and locates the centroid 55 km northwest of the $W$-phase centroid but still well within the aftershock cloud. This could reflect the GPS station distribution, all north of the rupture and across the U.S.–Mexico border. Nonetheless, comparison with the static slip inversion of Wei et al. (2011) shows that the $W$-phase solution is 25 km southeast of the main slip patch while our rapid solution lies within it. Similarly, the fastCMT solution favours a shallow centroid at a 4 km depth while the $W$-phase centroid is at 15 km depth. A shallow centroid at 2–8 km depth is consistent with the results of Wei et al. (2011). The moments of both the GCMT and fastCMT solutions are very similar at 6.8 and $6.0 \times 10^{21}$ Nm, respectively ($M_w$ 7.2). The strike and dip of the first nodal plane are similar for both solutions, although $W$ phase favours a small dip-slip component while the fastCMT solution has none.
Figure 9. (a) Maximum horizontal displacement and variance function used for determination of final offsets for the Tokachi-oki earthquake (see text for detailed description). $t_d$ is the time at which anomalous motion is detected, $t_{\text{max}}$ is the time of maximum variance, $t_1$ is the time when the variance function drops to 25 per cent of the maximum value and defines the start of the averaging interval and $t_2$ is 20 s after $t_1$ and is the end of the averaging interval and the time at which a final solution is available. (b)–(d) Final offsets determined for the three directions of motion for the Tokachi-oki event. (d) Maximum horizontal displacement and variance function used for determination of final offsets for the El Mayor-Cucapah earthquake (see text for detailed description); $t_d$, $t_{\text{max}}$, $t_1$, $t_2$ have the same definition as in (a). (f)–(h) Final offsets determined for the three directions of motion for the El Mayor-Cucapah event.

Figure 10. Detailed comparison between the (a) fastCMT and (b) GCMT results for the Tokachi-oki event. (c) Comparison between the best double-couple solutions as computed in (a) and (b) and the synthetic and observed displacements of (a).

The second nodal plane, which from the aftershock distribution is the actual fault plane, has similar strike for both solutions (318° and 319°, respectively) although the fastCMT solution favours a vertical fault plane while the $W$-phase nodal plane has a dip of 77°. Both solutions have similar rakes (217° and 213°) indicating predominant strike-slip motion with a significant dip-slip component consistent with regional transtensional tectonics (Hauksson et al. 2010). The largest difference between both solutions is in the size of the CLVD
component, the $W$-phase solution has a 10 per cent CLVD while the fastCMT solution has a 64 per cent CLVD. This seems unrealistically large for a normal tectonic earthquake; nonetheless other MT solutions exhibit large CLVD components as well. The GCMT solution (www.globalcmt.org) has CLVD of 52 per cent for this event while the Southern California Seismic Network’s solution (http://www.data.scec.org/) has a CLVD of 76 per cent. Furthermore, Hauksson et al. (2010) performed post-mortem $W$-phase inversions and show that a large CLVD is required. In that work the size of the CLVD is not explicitly stated but it ranges from 56 to 72 per cent (H. Kanamori personal communication, 2011). Thus the original $W$-phase estimate seems to underestimate the CLVD considerably while the fastCMT accurately assesses it for this particular event.

### 4. DISCUSSION

#### 4.1 Implementation issues

Thus far we have shown with the two test cases that obtaining rapid MT solutions and centroid locations from displacement waveforms is viable with our new technique. There are, however, a number of important issues that have to be addressed if fastCMT is to be implemented in real time.

A preliminary hypocentral determination needs to be made. In this study, because of the computational economy of the method where we can try thousands of different centroid locations and thus only a rough location estimate need be made, we opted for a self-contained rapid method that does not incorporate outside data and centres the inversion grid around the first stations to detect anomalous motion. As Crowell et al. (2009) showed, one can determine hypocentres with GPS. However given the greater sensitivity of traditional seismic instrumentation it would be more desirable to have weak motion and strong motion instrument-based methods emit a trigger and compute a hypocentre. Provided of course this can be done before time $t_1$ when the final offset estimation is made, ~3 min for the two events discussed earlier.

Another issue is whether to set the inversion nodes on a predefined set of fault surfaces, populate a rectangular prism of nodes around the hypocentre or have a more complex geometry of inversion nodes. For strike-slip environments having a library of fault surfaces and setting the inversion nodes on known geological surfaces close to the epicentre might seem appealing. However experience has shown that moderate-to-large events can occur on unknown faults and with dip-slip or oblique mechanisms. A good example of this is the 1994 $M_w$ 6.7 Northridge earthquake in southern California, which occurred on a previously unknown blind thrust fault. Thus, it may be more desirable to simply populate a prism of inversion nodes around the hypocentre.

For subduction zones it might be adequate to set the nodes on the plate interface. This would ensure correct centroid placement for traditional megathrust events, however outer rise, intraplate events on the overriding crust or in the downgoing slab, which can be sizable, may not be located properly. Evidently there are many ways to make these grids and the selected approach must be tailored to the observational goals of a particular network. In this paper we have adopted the simpler approach of defining a $3^\times3^\times1$ grid based on the location of the first five GPS stations that detect the event.

The speed of the inversion, the number of inversion nodes and the coarseness of the grid are also important considerations. Our inversion scheme is computationally efficient but the computing power required will depend on the number of inversion nodes. For reference each inversion using three component static offsets for 355 stations for the Tokachi-oki earthquake on one node every epoch takes about 75 ms with Matlab, on a single 800 MHz CPU with 2 GB of RAM. This will improve significantly when translated into a real-time amenable language and distributed over a cluster.

Another important point is that a GF library and the selected velocity models must be constructed before hand. Even though...
the functions are very succinct, because we can only accommodate 1-D velocity structure, the region over which the algorithm will be implemented must be adequately parsed. That is to say, based on the location of the desired inversion node and the station a suitable velocity model must be determined. In this case because we already knew the region of interest we selected a known velocity model for that region and applied it to all station–event paths. Nevertheless when building a large GF database the most sensible way to do this would be to select the average velocity model for each station–event path from a community model such as the CVM4. Additionally, in the Tokachi-oki case we simply neglected stations that experienced telemetry outages and did not register data long enough to extract a reliable offset. In a real-time implementation this problem is easily solved by ignoring that station, should an outage occur, and removing it from the GF matrix.

Finally, in terms of epistemic error there seem to be two important issues to consider. First whether neglecting fault finiteness has any impact on inversion results. Adamova & Sileny (2010) found that neglecting fault finiteness in waveform inversions can lead to spuriously large non-double-couple model parameters, however our Tokachi-oki inversion is almost pure double couple. Similarly the El Mayor-Cucapah result seems to require a large CLVD and that is most likely a real signal, thus it is not clear how large an error the point source assumption introduces in the inversions using static displacement data. Perhaps when the Tohoku-oki data become available this can be quantified further. Since we are employing local data it stands to reason that larger earthquakes might violate the point source assumption. Should that be the case incorporating higher order moments in the inversion (Adamova & Sileny 2010) should ameliorate the situation.

The second important source of error is the assumption of a 1-D earth. Amoroso et al. (2004) and Hearn & Bürgmann (2005) demonstrated the need for at least this simple approximation in inversions with coseismic offsets. Hinge et al. (2011) discuss both synthetic and observed results that indicate that waveform matching inversion techniques for MTs of medium sized events can be greatly affected by neglecting 3-D variations in the Earth structure. Nonetheless it is not clear in their results if the same is true for a method such as the one presented here that employs coseismic offsets. In any case, for the moment, it seems that for large events 1-D structure will suffice for rapid results.

4.2 Further improvements

From the basic MT information one could go a step further in real-time modelling of the source. Allen & Ziv (2011) performed an Okada-type slip inversion using the magnitude of the horizontal offsets, however their approach is limited since it necessitates a priori knowledge of the fault plane geometry. In this sense the information provided by the CMT computation method shown here can be used as input into these types of inversions. Even though the results obtained for the El Mayor-Cucapah earthquake are not as stable as those for the Tokachi-oki earthquake, the strike and dip information are still reliable throughout the inversion and can still be useful for rapid modelling, compared to the global CMT and post-processed solutions. We are currently exploring using strike and dip information from our MT inversion to seed further Okada-type inversions on an epoch by epoch basis, with encouraging results. Since both the Okada-type inversions as well as the CMT inversion discussed here are computationally efficient, one can run multiple iterations. For example one could run fault planes of different sizes on both nodal planes to distinguish between the rupture plane and the auxiliary plane, as well as to determine gross fault dimensions.

In our implementation, the final CMT solution is quickly obtained after the end of the rupture process, as soon as the full permanent deformation has been discerned. We took an empirical approach for the case when real-time displacements are available from near-source GPS networks. There are several ways in which the fastCMT process can be further enhanced in terms of speed and accuracy. We have opted for a very conservative 120 s moving average window. Crowell et al. (2009) considered scaling relationships between peak ground displacements (PGD) and the final static offsets suggesting that it might be possible to rapidly estimate coseismic offsets from such relations, thus increasing the timeliness of the rapid CMT solution. Allen & Ziv (2011) employed an short-term average/long-term average-type algorithm and were able to determine coseismic offsets for the El Mayor-Cucapah earthquake shortly after S-wave arrivals. Achieving faster and more robust estimates of static offsets is still an area of active research.

In this paper we have relied solely on GPS observations, which are much less precise than seismic data. This is most likely the result of poor constraints on the vertical component of displacement and residual noise in the horizontal components. This not only hinders the inversion results but poses a lower bound on the offsets that can reliably be detected (we set a threshold of 15 mm in this study) and limits the number of stations that can be used in modelling; this speaks directly to the lower magnitude bound of earthquakes that can be modelled with this approach. Bock et al. (2011) showed that one can combine GPS and accelerometer time-series in near real time using a Kalman smoother to compute broad-band velocity and
displacement time-series. The combination of these two data types also reduces the noise in the displacement time-series significantly, to the point where millimetre level displacements can be accurately measured such that one can even observe $P$ waves in the vertical component of displacement records at local to regional distances. As efforts to collocate strong motion sensors and GPS stations continue, the reliability and accuracy of displacement time-series and hence of the inversions they produce should improve.

A clarification must be made, the inversion results shown here even if performed each second are still static results and cannot resolve the evolution of rupture. Broad-band displacements as created by Bock et al. (2011) contain both the static offset and dynamic motions in a single time-series and will be very useful for the next step in rapid modelling: real-time kinematic and dynamic source inversions. Still we can say that GPS data are a unique source of information that allow us to better and more rapidly assess large events. Of course, this must be leveraged with traditional seismological instrumentation and techniques, and it will be the combination of all sources of data that will provide the best results.

These results are encouraging and data from more events are necessary to calibrate and benchmark the methodology discussed here. GEONET has recorded 29 $M_w > 7.0$ events in Japan since 2002. These data will help resolve many of the important issues discussed so far, and as they become more widely available will prove to be an invaluable source of information to better assess the role of GPS in rapid source modelling.

The fastCMT method requires a dense GPS network. These are only available in a few places around the Earth. We expect that projects such as this one and the ones cited throughout the text serve as further examples of the important role that real-time GPS can play in seismic monitoring and hazard mitigation and also to motivate seeding new networks and the continued expansion of existing ones.

CONCLUSIONS

We have presented an approach for real-time computation of centroid MTs for large events from local and regional GPS displacement records. We used the coseismic offsets and GFs for a layered earth to relate the deformation at the surface with source parameters at depth. We have demonstrated the algorithm with two test cases, the 2010 $M_w 7.2$ El Mayor-Cucapah earthquake and the 2003 $M_w 8.3$ Tokachi-oki earthquake. In both cases we have shown that provided with low latency access to displacement data it is feasible to obtain a robust centroid location and MT solution within the first 2–3 min after rupture initiation. The algorithm is computationally efficient and thus amenable for rapid modelling and early detection.

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The fastCMT code developed for these inversions can be downloaded from http://igpppublic.ucsd.edu/~melgar/CMT. We are indebted to Hrvoje Tkalčić and an anonymous reviewer for very detailed and constructive criticisms, which indubitably improved the content and presentation of the manuscript. This paper was funded by NASA/AIST Grant No. NNX09AI67G.

REFERENCES


Earthquake determined from tsunami arrival times at offshore observation stations, Earth Planets Space, 63, 809–813.

APPENDIX: EXTRACTING GF DATA FROM EDGRN

EDGRN output consists of three GF text files labelled ’ss’ (strike-slip), ’ds’ (dip-slip) and ’cl’ (CLVD), each one of these files is a look-up table consisting of the value of the GF for all three components of motion (as well as six components of strain and one tilt) at the discretized source–receiver distances and source depths. One peculiarity of this output is that the GFs are only computed for a line of receivers oriented due north (positive x-direction) from the source since GFs have an azimuthal dependence. The azimuthal relations are only mentioned in passing in Wang et al. (2003). Here we present the explicit expressions.

The output of EDGRN is in the radial (r), transverse (t) and vertical (z) coordinate system such that for each of the three components of displacement at any given station there are three GFs (consisting of a single number), that is, $H_{rs}$, $H_{rt}$ and $H_{rz}$ for the radial component, where $ss$ is the strike-slip component, $ds$ the dip-slip component and clvd the CLVD component. For the transverse direction of motion the GFs are $H_{ts}$, $H_{tt}$ and $H_{tz}$. Thus, for each direction of motion there will be five GFs, one for each component of the MT (eqs A1–A3). Then, for the $k$th station where $\phi_k$ is the source to receiver azimuth we have for the radial direction of motion

$$H_{rs} = H_{rs} \sin 2\phi_k$$
$$H_{rt} = H_{rt} \cos \phi_k$$
$$H_{rz} = H_{rz}$$

(A1)

for the transverse direction

$$H_{ts} = H_{ts} \cos 2\phi_k$$
$$H_{tt} = H_{tt} \sin \phi_k$$
$$H_{tz} = 0$$

(A2)

and for the vertical direction

$$H_{ts} = H_{ts} \sin 2\phi_k$$
$$H_{tt} = H_{tt} \cos \phi_k$$
$$H_{tz} = H_{tz}$$

(A3)

The matrix equation can then be set up as

$$v_j^R = \begin{pmatrix} H_{rj}^1 & H_{rj}^2 & H_{rj}^3 & H_{rj}^4 & H_{rj}^5 \\ H_{tj}^1 & H_{tj}^2 & H_{tj}^3 & H_{tj}^4 & H_{tj}^5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_{nj}^1 & H_{nj}^2 & H_{nj}^3 & H_{nj}^4 & H_{nj}^5 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_5 \end{pmatrix},$$

(A4)

or more succinctly

$$v_j^k = H_{jk}^i m_j; \ {i = r, t, z; \ j = 1, 2, \ldots, 5; \ k = 1, 2, \ldots, n},$$

(A5)

where as before $v$ contains the observed displacements, $H$ the GFs and $m$ is the MT in vector form. The actual input data however are in $(x, y, z)$ coordinates, thus we build a block diagonal
rotation matrix

\[ R = \begin{bmatrix}
\cos \phi_1 & -\sin \phi_1 & 0 \\
\sin \phi_1 & \cos \phi_1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\cos \phi_k & -\sin \phi_k & 0 \\
\sin \phi_k & \cos \phi_k & 0 \\
0 & 0 & 1
\end{bmatrix} \]  \hspace{1cm} (A6)

where once again the \( \phi_k \) corresponds to the individual source–receiver azimuths. Then, the final matrix equation that is to be inverted is

\[ u = RHm = Gm. \]  \hspace{1cm} (A7)

**SUPPORTING INFORMATION**

Additional Supporting Information may be found in the online version of this article:

**Movie S1.** Movie showing the results for the full 300 s Tokachi-oki inversion. The times depicted are seconds after origin.

**Movie S2.** Movie showing the results for the full 300 s El Mayor-Cucapah inversion. The times depicted are seconds after origin time.

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