REFERENCE POINT EQUALIZATION METHOD FOR DETERMINING THE SOURCE AND PATH EFFECTS OF SURFACE WAVES

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Abstract. A reference point equalization method is presented for the determination of source and propagation effects of surface waves. The method works on seismic events located in a small source region, allowing the assumption that all events share the same path effects to a given receiver. Two steps in the method are initialization and iteration. Initialization obtains the first reference events in order to compute initial estimates of phase velocity and attenuation coefficient. Iteration simultaneously refines the propagation parameters and determines source parameters of new earthquakes. This method was applied to nine earthquakes in the Pamir Mountains, Central Asia (reference point: 39.58°N, 73.55°E). Source parameters were determined using the linear moment tensor inversion on 26 to 60-s Rayleigh wave complex spectra. It was necessary to modify the straight line squares inversion method because of its sensitivity to even just a few bad data points in the data set. Residuals obtained from repeated application of the moment tensor inversion over trial focal depths showed two minima: one at depths less than 20 km and the other at depths greater than 70 km, with values of the residuals at these minima close enough to cast doubt on the determination of focal depth. The inversion generally gave three-couple force systems having significant nonzero intermediate component. These features of source parameter inversions are discussed in light of errors in the data set. Propagation parameters, in particular, the phase velocities, show good qualitative correlation with geologic and geographic features on the Eurasian continent.

1. Introduction

One advantage of the seismic moment tensor formalism obtained by Gilbert [1970] is that the amplitude of free oscillations is linearly related to source parameters, in this case, the moment tensor elements. Because of the speed of linear inversion techniques this formalism raises the possibility of making routine determinations of earthquake source mechanisms like hypocenter locations are done today. The extension of this formalism to other data sets has been proposed by a number of investigators [Buland and Gilbert, 1976; McCowan, 1976; Stump and Johnson, 1977; Mendiguren, 1977]. Assuming a point source in a layered medium, Mendiguren expressed the surface wave complex spectrum as a linear combination of six moment tensor elements. Aki and Patton [1978] have shown that the use of surface wave complex spectra in a period range recorded by the World-Wide Standard Seismograph Network (WWSSN) requires accurate knowledge of the path effects, particularly the phase velocity. For example, to obtain the initial phase of 30-s Rayleigh waves within ±0.1 cycles, the phase velocity on a 2000-km path must be known to 0.5%. Longer paths or shorter-period waves will demand even greater accuracy.

This paper describes a reference point equalization method for separating the source and path effects of surface waves. The method works on seismic events located in a small source region, which allows us to assume that all events share the same path effects to a given receiver. Two important steps in the method are initialization and iteration. The latter refines estimates of the path parameters as the mechanisms of seismic events are determined. The path parameters for teleseismic paths as long as 7000 km are sufficiently accurate to isolate the complex source spectrum of 25 to 60-s Rayleigh waves. This paper discusses an application to Central Asian earthquakes and gives the results of the analysis including moment tensor inversions.

2. Method

The method requires seismic events in a small area with a lateral dimension, D, much smaller than the epicentral distance and comparable to or less than the shortest analyzed wavelength. This requirement allows us to assume safely that all events in the source region share the same wave path to a given receiver except for small differences in the path length in the source region. These differences will be corrected for by introducing a reference point as shown schematically in Figure 1.

The reference point is defined as the point with coordinates equal to the mean of coordinates of all the events. Then the distance \( X_i \) from the reference point to the ith receiver can be written as

\[
X_i = \frac{1}{N} \sum_{j=1}^{N} X_{ij}
\]

where \( X_{ij} \) is the epicentral distance between the jth event and the ith station and \( N \) is the number of events in the source region. The effect of the correction will be to equalize the Fourier transform of the surface wave record to \( f_1 \). Specifically, we compute the Fourier transform of a record, \( f_1(t) \), over a time interval \( t_1 \) to \( t_2 \), as follows:
Fig. 1. Schematic of the source region having lateral dimension $D$; $\lambda$ is the shortest analyzed wavelength, and $X_{ij}$ is the epicentral distance between the $j$th earthquake and the $i$th receiver.

$$\Theta_{jik}^O - i\phi_{jik}^O = \Theta_{j1}^O(\omega_k) e^{-i\phi_{j1}^O(\omega_k)}$$

$$= \int_{t_1}^{t_2} \Theta_{j1}(t) e^{-i\omega_k t} dt$$

where $k$ refers to frequency $\omega_k$, $\Theta_{jik}^O$ is the amplitude spectral density, and $\phi_{jik}^O$ is phase delay in cycles. The observed amplitude and phase spectra are equalized to the reference point as follows:

$$A_{jik}^E = A_{jik}^O$$

and

$$\phi_{jik}^E = \phi_{jik}^O + \frac{\omega_k(X_j - X_{ij})}{C(\omega_k)}$$

where the superscript $E$ indicates the equalized quantity. Since path differences are small, the effects of attenuation are ignored. Phase equalization is made using a phase velocity curve, $C(\omega_k)$, appropriate for the source region.

After correction for instrumental response, the equalized quantities $A_{jik}^E$ and $\phi_{jik}^E$ are separated into source and path effects:

$$A_{jik}^E = H_{ik}(\omega_k) S_{ji}^O(\omega_k) = H_{ik} S_{ji}^O$$

$$\phi_{jik}^E + \omega_k T_{ji} = \phi_{ik}^O + \phi_{ji}^F(\omega_k) + n$$

where $S_{ji}^O$ is the source amplitude factor and $\phi_{ji}^F$ is the source or focal phase delay. This is the phase of the Rayleigh wave at the origin time and epicenter of the earthquake and is sometimes referred to as the far-field first motion of the Rayleigh wave [e.g., Weidner, 1972]. $H_{ik}$ is the path amplitude transfer function which corrects for anelastic attenuation and geometric spreading, and $\phi_{ik}$ is the propagation phase delay. $T_{ij}$ refers to the time interval between the origin time of the $j$th earthquake and the start of the digitization window, $t_1$, on the $i$th record. The integer $n$ represents the order number [Brune et al., 1960].

Earthquakes in the source region with known source parameters (i.e., fault plane geometry, depth, seismic moment) shall be called reference events. For reference events we can calculate $S_{ji}^O$ and $\phi_{ji}^F$ using the theory of surface wave excitation [e.g., Saito, 1967]. Thus if there are $N$ reference events, we obtain, by (4), $2N$ equations at each frequency which relate source parameters with the observed spectra at the $i$th station through a common earth filter parameterized by $H_{ik}$ and $\phi_{ik}$. The errors in the observations and assumptions in our model will require a statistical method to estimate these filter parameters. Let us call $\tilde{H}_{ik}$ and $\tilde{\phi}_{ik}$ the estimates of the filter parameters determined by an appropriate statistical method.

If a new earthquake, say the $k$th, does not have known source parameters, the estimates, $\tilde{H}_{ik}$ and $\tilde{\phi}_{ik}$, will be used to isolate its source spectra by the following equation:

$$A_{jik}^E e^{-i\phi_{jik}^E} = E \frac{-i\phi_{jik}^E}{\tilde{H}_{ik} e^{-i\phi_{ik}}}$$

where $A_{jik}^E$ is identified as the source amplitude factor and $\phi_{jik}^E$ as the focal phase delay of the $k$th earthquake. Source spectra obtained in this manner are used to recover the source parameters of this new earthquake.

The reference point equalization allows the separation of path and source effects for earthquakes in the source region, provided that reference events are available to compute the propagation parameters. Initial estimates of $H_{ik}$ and $\phi_{ik}$ are obtained from a method that does not require reference events or are based on a priori knowledge of the propagation effects.

Initialization via Weidner and Aki

The method of Weidner and Aki [1973] requires two earthquakes located close together and having different focal mechanisms. Forming spectral ratios between earthquakes at a given receiver will cancel the propagation effects but not the source effects if the two earthquakes have different focal mechanisms. Under these conditions the ratios can be used to revise their source parameters.

The source parameters of both earthquakes are revised to minimize the residuals between the observed log amplitude ratio, $\ln(A_{jik}^O/A_{2ik}^O)$,
Fig. 2. Azimuthal equidistance projection of Eurasia centered on the reference point in the Pamir Mountains. All stations used in this study are shown on this map.

and the calculated, as well as between the observed differential phase, $\Delta \theta_{1-2,ik}$:

$$\Delta \theta_{1-2,ik} = \theta_{1ik} - \theta_{2ik}$$

and its calculated counterpart. The residuals are simply a weighted sum of squared differences with the weights chosen to minimize the contribution from stations near the nodal directions. A systematic search through an eight-dimensional parameter space (two depths, two slips, two dips and two strikes) is carried out to find source parameters that minimize the residuals.

By virtue of the cancellation of propagation effects, a priori knowledge of $H_{ik}$ and $\phi_{ik}$ is not required. The source region structure must be known, though errors in the phase velocity curve $C(\omega)$ due to poor knowledge of this structure will not significantly affect the calculation of $\Delta \theta^0$ because of the small distances between events.

The source parameters that minimized the residuals are used to calculate initial estimates of $H_{ik}$ and $\phi_{ik}$.

Iterative process

The iterative process starts after initial estimates of $H_{ik}$ and $\phi_{ik}$ are determined. Once a new event has been introduced and its source parameters determined, this event is placed in the pool of reference events to be used in revising $H_{ik}$ and $\phi_{ik}$. The revision that occurs in each iteration results in refined estimates of these path parameters. Finally, with improved estimates of the path parameters, it should be possible to revise the source parameters of some events in the pool.

We anticipate that the largest instabilities will arise at the outset when estimates of $H_{ik}$ and $\phi_{ik}$ are based on just a few observations. A fully developed reference point will result once $H_{ik}$ and $\phi_{ik}$ have converged, implying that the addition of new events and the revision of pooled events will cause little change in these estimates.

After full development we move to a new reference point in a nearby source region, and thus the supply of new events is endless. The movement takes place by a small jump from the initial reference point along the worldwide seismic belts. The next reference point is located in a source region with some overlap of the previous region. In this manner, reference point 2 and onward will always have the necessary reference events with which to start the iteration.
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Table 1. Epicentral Data From ISC

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Origin Time, UT</th>
<th>ΔO.T., s</th>
<th>Latitude, deg N</th>
<th>ΔLatitude, deg</th>
<th>Longitude, deg E</th>
<th>ΔLongitude, deg</th>
<th>Depth, km</th>
<th>( m_b )</th>
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<tbody>
<tr>
<td>1</td>
<td>May 11, 1967</td>
<td>1450:57</td>
<td>+1.80</td>
<td>39.33</td>
<td>+0.028</td>
<td>73.74</td>
<td>+0.031</td>
<td>2</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>Aug 28, 1969</td>
<td>0358:36.7</td>
<td>+0.20</td>
<td>39.07</td>
<td>+0.032</td>
<td>73.61</td>
<td>+0.042</td>
<td>22</td>
<td>5.2</td>
</tr>
<tr>
<td>3</td>
<td>Sep 14, 1969</td>
<td>1615:25.6</td>
<td>+0.57</td>
<td>39.70</td>
<td>+0.022</td>
<td>74.80</td>
<td>+0.025</td>
<td>38</td>
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</tr>
<tr>
<td>4</td>
<td>Jul 24, 1971</td>
<td>1143:39.3</td>
<td>+0.40</td>
<td>39.47</td>
<td>+0.019</td>
<td>73.18</td>
<td>+0.024</td>
<td>36</td>
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</tr>
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<td>5</td>
<td>Oct 28, 1971</td>
<td>1330:56.4</td>
<td>+0.92</td>
<td>41.88</td>
<td>+0.020</td>
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<tr>
<td>6</td>
<td>Nov 12, 1972</td>
<td>1756:52.9</td>
<td>+0.25</td>
<td>38.33</td>
<td>+0.016</td>
<td>73.17</td>
<td>+0.017</td>
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<tr>
<td>7</td>
<td>Aug 11, 1974</td>
<td>2005:30.9</td>
<td>+0.30</td>
<td>39.44</td>
<td>+0.014</td>
<td>73.67</td>
<td>+0.016</td>
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<tr>
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<td>Aug 11, 1974</td>
<td>2121:37.1</td>
<td>+0.85</td>
<td>39.46</td>
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<td>+0.020</td>
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<tr>
<td>9</td>
<td>Aug 27, 1974</td>
<td>1256:01.0</td>
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<td>+0.021</td>
<td>73.82</td>
<td>+0.024</td>
<td>19</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Recovery of source parameters

Our observations consist of complex source spectra obtained in (5), where \( \phi_{ik} \) is the amplitude at the frequency \( \omega_k \) in the azimuth of the \( i \)th station at a great-circle distance \( \chi_i \) from the source. (The subscript \( g \) has been dropped to simplify notation.) Assuming the geometrical spreading factor for a laterally homogeneous spherical earth, the amplitude is reduced to the amplitude of the distance-independent quantity \( S \) defined by (2) in the work of Patton and Aki [1979]. The phase delay \( \phi_{ik} \) will be reduced to the phase of \( S \) by removing the phase shift \(-\pi/4 \) coming from the asymptotic expansion of the Hankel function and the phase delay \( \pi/2 \) introduced by the slip time function, in this case assumed to be a step function. The real part \( \alpha_{ik} \) and the imaginary part \( \beta_{ik} \) of this reduced spectrum are related to the elements of the seismic moment tensor as follows:

\[
\alpha_{ik} = M_{zz} (G_{2k}(h)-G_{1k}(h)) - (M_{yy}-M_{xx})G_{1k}(h) \cos 2\theta_{ik} \\
+ 2M_{xy} G_{1k}(h) \sin 2\theta_{ik} + \alpha_{ik}
\]

\[
\beta_{ik} = M_{xz} G_{3k}(h) \cos \theta_{ik} + M_{yz} G_{3k}(h) \sin \theta_{ik} + \beta_{ik}
\]

where the tensor elements \( M_{ij} \) are given in Cartesian coordinates with origin at the source and the \( x, y, \) and \( z \) axes pointing east, north, and up, respectively. The \( G_{ik} \), also defined by Patton and Aki, are the medium responses of the Rayleigh wave in a vertically heterogeneous medium, \( h \) is the depth of the point source below the surface, \( \theta_i \) is the receiver azimuth measured counterclockwise from east, and the \( \phi_{ik} \) are error terms. In deriving (7) we have constrained \( M_{xx}+M_{yy}+M_{zz}=0 \).

The source depth, which is the only nonlinear source parameter that remains in (7), must be determined by repetitive application of the linear inversion. At each trial depth a sum of squared residuals will be computed by the following formula:

\[
\epsilon^2 = \sum \left( (\alpha_{ik}^2 + \beta_{ik}^2) \right)
\]

The depth which minimizes this residual will be chosen as the source depth. The moment tensor is calculated directly from the linear inversion carried out at this chosen depth.

There are errors that do not enter linearly on the complex spectrum, as assumed in (7). For example, when the noise is primarily signal-generated, as in the case of focusing and multi-path interference, errors will enter additively on the log amplitude and phase spectrum [Pilant and Knopoff, 1964; Aki, 1973] instead of the real and imaginary parts of the complex spectrum. Consequently, the estimation of source parameters should be based on minimizing the following error expression:

\[
\epsilon^2 = \sum \left( (\ln \alpha_{ik}^2 + \ln \beta_{ik}^2 + (\phi_{ik}^2 + \phi_{ik}^2)) \right)
\]
where $M_0$ is the seismic moment, $A_{1k}$ is the source amplitude, computed for a unit moment double couple, and $\phi_{1k}$ is the computed focal phase. It is understood that

$$A_{1k} = A(h, s, d, \theta^F - \theta_i, \omega_k)$$  \hspace{1cm} (10)

$$\phi_{1k} = \phi(h, s, d, \theta^F - \theta_i, \omega_k)$$

where $h$ is the source depth, $s$ is the slip angle, $d$ is the dip angle, $\theta^F$ is the strike of the fault, and $\theta_i$ is the station azimuth. Unfortunately, the log amplitude and phase are not linearly related to the source parameters. Furthermore, a linearization scheme would encounter difficulties because of the nature of the logarithm and arctangent. The minimization of the quantity $\sigma^2$ requires a systematic search through parameter space as is done in the method of Weidner and Aki [1973]. The moment $M_0$ is estimated by minimizing the error in amplitude for the trial mechanism, i.e.,

$$\min \varepsilon^2 = \sum_{i,k} \left[ (\ln A_{1k} - \ln M_0 A'_{1k})^2 \right]$$  \hspace{1cm} (11)

where $A_{1k}'$ is the trial amplitude. If there are a total of $M$ observations, then we compute the estimate $M_0$ as follows:

$$\ln M_0 = \frac{1}{M} \sum_{i,k} (\ln A_{1k} - \ln A_{1k}').$$  \hspace{1cm} (12)

Error minimization for computing $\hat{A}$ and $\hat{\phi}$

In the case of signal-generated noises an error expression in log amplitude and phase similar to (9) may be found for computing estimates of the path parameters [Patton, 1978].

Error minimization of the expression leads to the following formulae:

$$\ln \hat{M} = \frac{1}{M} \sum_{j=1}^{M} \ln H_j$$  \hspace{1cm} (13)

$$\hat{\phi} = \frac{1}{M} \sum_{j=1}^{M} \phi_j$$

Fig. 3. Fundamental mode Rayleigh wave responses as defined by Patton and Aki [1979] for six focal depths (kilometers) in the Pamir earth model. $G_i$ are defined by Patton and Aki [1979], $k$ is wave number, $U$ is group velocity, and $\omega^2 I_1$ is kinetic energy of the surface wave.

Fig. 4. P wave first motions plotted on a stereographic net: solid circles, compression; open circles, dilation; crosses, no P wave; solid circles with crosses, uncertain compression; open circles with crosses, uncertain dilation. Fault plane solutions for events 1 and 3 were obtained by Molnar et al. [1973].
where $M$ is the number of events observed at the $i$th station and the frequency dependence of these parameters is understood. These formulae give maximum likelihood estimates for $\ln H$ and $\phi$ when $\ln H_j$ and $\phi_j$ follow a Gaussian distribution.

When the source of noise is background recording noises, the error minimization for the path parameters is expressable in terms of the real and imaginary parts of the surface wave spectra. Assuming the errors in the complex spectrum obey Gaussian statistics, Pisarenko [1970] found the following formulae:

$$\hat{H} = \frac{V-zU}{2(V^2+x^2)} + \sqrt{\frac{(V-zU)^2}{4(V^2+x^2)} + z}$$  \hspace{1cm} (14)

$$\hat{\phi} = \tan^{-1}\left(\frac{x}{w}\right)$$

where

$$U = \frac{1}{M} \sum_{j=1}^{M} (cF_j + sF_j)$$

$$V = \frac{1}{M} \sum_{j=1}^{M} (cF_j - sF_j)$$

$$W = \frac{1}{M} \sum_{j=1}^{M} (cF_j sF_j + sF_j cF_j)$$

$$X = \frac{1}{M} \sum_{j=1}^{M} (cF_j sF_j - sF_j cF_j)$$

and $(cF_j, sF_j)$ are the real and imaginary part of the input or source spectrum and similarly $(cF_j, sF_j)$ for the output or observed spectrum at the $i$th station (index dropped for simple notation) of the $j$th event. The parameter $z$ is the ratio of noise variance on the output to noise variance on the input. The formulae in (14) are maximum likelihood estimates of the path parameters when errors in the complex spectra follow Gaussian statistics.

3. Data

The data required in the analysis consist of epicentral data on the earthquakes in the source region, digitized seismograms over the time window for surface waves, and body wave data in the form of body wave fault plane solutions required for the initialization. The source region selected for this study is located in Central Asia, north of the Pamir thrust zone. As shown by the map centered at the reference point (Figure 2), this location has excellent coverage by approximately 50 stations of the WWSSN. The surface wave data cover most of the landforms found on the Eurasian continent. The average path length is approximately 65-70 km under the Pamir Mountains south of the source region. Along a profile between the Tien Shan, just north of the reference point, and the Pamir Mountains [Kosminskaya and Riznichenko, 1964] the crust was interpreted to have two layers, each 30 km thick, the upper and lower layers having P wave velocities of 5.5 and 6.5 km/s, respectively. This profile is incorporated into the model for the source region structure given in Table 2. The crust is overlain by a sedimentary layer based on results of Arkhangel'skaya and Kuznestsova [1969], Molnar et al. [1973], and Chen and Molnar [1975]. The upper mantle structure is assumed to be the same as the Gutenberg earth model. Using a flat earth approximation, we computed the phase velocity dispersion curve and the medium responses shown in Figure 3. The dispersion profile agrees well with observed phase velocities in the source region [Savarensky et al., 1969] and is used in the calculation of $\phi E$ in (3).

P wave fault plane solutions

Body wave data in the form of P wave solutions is required in the initialization step using the earthquake pair method. The two solutions (events 1 and 3) shown in Figure 4 were obtained by Molnar et al. [1973] and satisfy the requirement that the earthquakes have different focal mechanisms.

Surface wave data

The surface wave data set was obtained from the vertical component seismograms of the WWSSN shown in Figure 2. A time window $(t_1-t_2)$ that contained the fundamental mode Rayleigh wave was determined for each record. The time of the start of the window, $t_1$, was estimated assuming a group velocity of 4.2 km/s. The end of the window, $t_2$, corresponded in most cases to velocities in the range of 2.0-2.6 km/s.
Fig. 5. (a, b) Wave forms and the results of the moving window analysis. Values of group velocity are shown at minute marks along the wave forms. Energy contours are shown at 4 and 8 dB down from the maximum computed at each period.
In addition, all records were filtered using the time-variable filter technique [Landisman et al., 1969]. This involved two processing steps. First, group velocities were obtained by the moving window analysis. The output of this analysis gives a two-dimensional plot (velocity versus period) of the energy contained in a windowed portion of the time series. The position and length of the cosine-squared window, applied to the time series, depend on the velocity and period to be analyzed. In our case the length was always set to 4 times the period. The arrival of the group or wave packet is inferred from the energy contours drawn on the resultant plot. The second step filters the seismogram by passing only those wave packets with group velocities that correspond to the fundamental mode Rayleigh wave. Weidner [1972] found that this filtering technique reduces noise without distorting the phase of the original record. The present author confirmed this by overlaying the filtered record on the original.

**Noise**

On many records, background noise samples of the same length as the signal window were digitized before and after the arrival of the signal. The noise time series were detrended, Fourier-transformed, and corrected for instrumental response in the same manner as the signals. The background noise levels in the frequency range 0.025–0.05 Hz were found to be well below signal levels on most records.

Signal-generated noise caused by body wave and higher-mode interference, focusing of the primary waves, and multipath interference can be expected to pose more serious problems in the linear inversion than background noises [Patton and Aki, 1979]. Strong excitation of higher modes is common for crustal earthquakes in Central Asia [Forsyth, 1976]. Propagation of higher modes in Eurasia [Crampin, 1966] is characterized by very efficient transmission across the stable platform of northern Asia and

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**Table 3. Source Parameters of Events 1 and 3 Obtained From Earthquake Pair Method of Weidner and Aki**

<table>
<thead>
<tr>
<th>Event 1</th>
<th>Event 3</th>
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<tbody>
<tr>
<td>Focal Depth, km</td>
<td>5</td>
</tr>
<tr>
<td>Slip angle, deg</td>
<td>40</td>
</tr>
<tr>
<td>Dip angle, deg</td>
<td>100</td>
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<tr>
<td>Fault strike, deg</td>
<td>215</td>
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<tr>
<td>Seismic moment, x10^{24} dyn cm</td>
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</tr>
<tr>
<td>M_{xx}</td>
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</tr>
<tr>
<td>M_{xy}</td>
<td>1.4</td>
</tr>
<tr>
<td>M_{yy}</td>
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Fig. 7a. Amplitude radiation patterns of event 9. Twofold symmetry is reflected about N-S axes, showing stations east of north as pluses and west of north as solid circles. Screened data points are shown by open circles. Shown are results of linear inversion on all data (dashed lines), on screened data (solid lines), and logarithmic fitting on all data (dot-dashed lines).

The effects of focusing and multipathing are expected to be more serious. Focusing and defocusing is the result of refraction of surface waves due to lateral variations in velocity along the wave path [McGarr, 1969]. Studies by Capon [1970] and Bungum and Capon [1974] concluded that the major cause of multipathing is lateral reflections of waves off continental margins, mountain chains, and midocean ridges. Thus the type of interference will depend on the frequency range and on the nature of the wave path between the source and receiver. On the Eurasian continent, for example, no multipathing is observed at Norsar for 40-s Rayleigh waves originating from the Lop Nor nuclear test site, 2000 km west of the reference point [Bungum and Capon, 1974]. The observations of 20-s Rayleigh waves from Lop Nor showed evidence of multipathing, but the results of their analysis may have been affected by the occurrence of an interfering earthquake. Figure 5 shows a few examples of wave forms used in this study and the results of the moving window analysis on these wave forms. While wave forms observed at northern European stations showed little interference, wave forms recorded by stations east and west of the reference point were complex. The contrast is strongest for stations across tectonic provinces of China and the European stations lying north of the Russian platform. Multipath arrivals, well separated from the primary arrival, as in the case of HKC in Figure 5, were eliminated by the time-variable filtering. Considering the lengths of the wave paths [Aki et al., 1972] and the complexities due to...
the noise sources discussed above, we were forced to restrict our data analysis to periods longer than 25 s.

4. Initialization

As described above, the method of Weidner and Aki [1973] requires two earthquakes having different P wave solutions. By virtue of their proximity the spectral ratios between these earthquakes will cancel the propagation effects to a given station and retain the effects of source differences. Events 1 and 3 in Table 1 were chosen for this analysis.

Following Weidner and Aki, residuals are defined by

$$\sigma^2_\phi = \frac{\sum_k (\Delta \phi^0_{1,3,k} - \Delta \phi^T_{1,3,k})^2}{\sum_k}$$

for phase data and by

$$\sigma^2_A = \frac{\sum_k (\ln(A^0_{3,1,k}/A^0_{3,1,k}) - \ln(A^T_{3,1,k}/A^T_{3,1,k}))^2}{\sum_k}$$

for amplitude data, where the superscripts 0 and T refer to observed and theoretical quantities, respectively, and $W_k$ is the weight. The remaining symbols have been defined above. The weight on each phase observation is equal to the average of the observed amplitudes, $(A^2_{3,1,k} + A^2_{3,1,k})/2$. In the case of the amplitude residual the weight is computed from the average of theoretical amplitudes, $(A^2_{3,1,k} + A^2_{3,1,k})/2$ in order to minimize the contribution from stations lying in the node of the radiation pattern.

A trial and error search of parameter space is carried out to find the depth and mechanism of each event that minimizes the residuals. The search does not cover the entire eight dimensional space, as constraints are imposed by the P wave solutions. Figure 6 shows plots of the residuals versus source parameters in the vicinity of the point in parameter space giving a minimum in the residuals. The curves in each box in this figure were obtained by holding all of the parameters fixed at the values which gave a minimum and varying the parameter specified for the box.

The resultant plots show good agreement between the amplitude and phase data sets. The solutions for both data sets are located at the absolute minimum in the residuals, and no distinct local minima were found that suggested other possible mechanisms. Weidner [1972] found that the amplitude residual at depths of 50-60 km was about equal if not smaller than the residual at shallow depths for the Mid-Atlantic Ridge earthquakes. The phase residual was large for the deep focus solution, however, and thus resolved the ambiguity. In our search over the range of 0-55 km we do not see this ambiguity in the amplitude residual. Indeed, the search converged directly to the solution shown above. Some indication of the good definition of this minimum is given by the slices of parameter space shown in each box in Figure 6.
Seismic moments

In fitting amplitude ratios the analysis above gives an estimate of the ratio of the seismic moments between events 1 and 3. Rather than assume an initial Q model for Eurasia and risk introducing a bias that would remain in later Q models, the moments and attenuation coefficients were estimated simultaneously by the method of Tsai and Aki [1969].

The observed amplitudes at a given frequency were corrected for the radiation pattern computed from the source parameters obtained above and for geometric spreading. The resultant amplitudes were plotted on a log scale as a function of distance from the source. The slope of this plot is proportional to the attenuation coefficient of the seismic waves, while the intercept gives an estimate of the moment.

The source parameters for events 1 and 3 are summarized in Table 3.

5. Iteration

Once initialized, the method follows the iterative cycle described above. We estimate the path transfer functions for those stations that recorded reference events 1 and 3. At long periods (T > 40 s) the estimates of H and $\phi$ are calculated using (14) based on Gaussian random errors in the complex spectrum and assuming $z = 1.0$. At short periods the estimates are calculated using (13).

All nine events in Table 1 were introduced one by one (not necessarily in numerical order) into the iterative cycle. The estimates of H and $\phi$ and the source parameters of events 1 and 3 were subsequently revised in this process. As an
example, the results of a detailed analysis of event 9 are given below.

Figures 7a and 7b show radiation patterns of the source amplitude $A^s$ and source phase $\delta^s$, respectively, for event 9. The amplitude radiation pattern is plotted over $180^\circ$ by making use of its twofold symmetry. Phase is plotted in radians over $360^\circ$ of azimuth. In Figures 8a and 8b we have plotted the real and imaginary parts corresponding to $a_{ik}$ and $b_{ik}$ defined in (7). The radiation patterns of the real part in Figure 8a show the expected $\sin 2\theta$ dependence, and those of the imaginary part of Figure 8b show the expected $\sin \theta$ dependence. Only the data shown in these figures were included in the analysis described below.

The results of three experiments, (1) linear inversion on all of the data shown in Figure 8, (2) linear inversion with a few omitted data points shown by open circles in Figure 8, and (3) trial and error search in logarithm space using (9) to compute the residual, have been discussed in an earlier paper [Aki and Patton, 1978] and are summarized here in Table 4. The agreement between the results from the logarithmic fit and the linear inversions is very good. It is notable that the biggest differences between the calculated radiation patterns plotted in Figures 7 and 8 occur where the station coverage is poorest. At short periods the logarithmic fit favors the result obtained from the linear inversion on the data set which included erratic data points. Apparently, these points did not affect the results of the trial and error method because their influence is unnoticed in the logarithmic residual space. This demonstrates a distinct advantage of a robust method, such as the logarithmic fit, over the
Table 4. Results of Linear Inversions and Logarithmic Fitting Applied to Event 9

<table>
<thead>
<tr>
<th>Method</th>
<th>Linear Least Squares Inversion, All Data</th>
<th>Linear Least Squares Inversion, Screened Data</th>
<th>Trial and Error Fit, lnA+i, All Data</th>
<th>Robust Inversion, All Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Depth, km</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Seismic moment, x10^24 (dyn cm)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>(M_{zz})</td>
<td>-0.1 ± 0.6</td>
<td>-0.3 ± 0.5</td>
<td>0.4</td>
<td>0.4 ± 0.3</td>
</tr>
<tr>
<td>(M_{xy} - M_{xx})</td>
<td>-4.8 ± 0.9</td>
<td>-4.8 ± 0.8</td>
<td>-6.6</td>
<td>-4.8 ± 0.3</td>
</tr>
<tr>
<td>(M_{yz})</td>
<td>3.1 ± 0.4</td>
<td>3.1 ± 0.4</td>
<td>3.0</td>
<td>2.8 ± 0.2</td>
</tr>
<tr>
<td>(M_{xz})</td>
<td>0.3 ± 0.2</td>
<td>0.9 ± 0.2</td>
<td>1.3</td>
<td>0.8 ± 0.1</td>
</tr>
<tr>
<td>(\lambda_1^*)</td>
<td>1.2 ± 0.2</td>
<td>2.3 ± 0.2</td>
<td>2.2</td>
<td>1.8 ± 0.2</td>
</tr>
<tr>
<td>(\lambda_2^*)</td>
<td>4.1 ± 0.5</td>
<td>4.8 ± 0.5</td>
<td>5.2</td>
<td>4.1 ± 0.2</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>0.0 ± 0.6</td>
<td>-0.3 ± 0.4</td>
<td>0.0</td>
<td>0.2 ± 0.3</td>
</tr>
<tr>
<td>(e_1^+)</td>
<td>-1170</td>
<td>-1210</td>
<td>-1160</td>
<td>-1180</td>
</tr>
<tr>
<td>(e_2)</td>
<td>110</td>
<td>210</td>
<td>240</td>
<td>230</td>
</tr>
<tr>
<td>(e_3)</td>
<td>-2650</td>
<td>-2680</td>
<td>-2560</td>
<td>-2580</td>
</tr>
<tr>
<td>(e_4)</td>
<td>720</td>
<td>580</td>
<td>600</td>
<td>610</td>
</tr>
<tr>
<td>(e_5)</td>
<td>-240</td>
<td>-210</td>
<td>-180</td>
<td>-210</td>
</tr>
<tr>
<td>(e_6)</td>
<td>160</td>
<td>240</td>
<td>190</td>
<td>170</td>
</tr>
</tbody>
</table>

* Eigenvalues of the seismic moment tensor.

+ Orientation of principal axes given by azimuth from north (positive: clockwise) and dip angle from horizontal plane.

least squares inversion when outliers are present. Robust methods applicable to linear inversions have been researched extensively by many statisticians (see Huber [1972] for a review) and some of these methods have been applied to geophysical data [Claerbout and Muir, 1973]. Table 4 shows the results of a robust inversion employing weighted, iterative least squares, the details of which are outside the discussion of this paper. The robust inversion is explicitly described by Patton [1978]. It should be noted that introducing robust methods into the linear inversion increases the computational time over straight least squares. Nevertheless, the trial and error search method in logarithmic space is still an order of magnitude slower than the robust linear inversion.

**Residuals as a function of focal depth**

We have computed the residuals of the robust linear inversion method over a wide range of depths in Figure 9. The residual curve shows a clear minimum at about 15 km. A remarkable feature of the curve is the occurrence of another minimum centered at about 90 km. The residual at this minimum is only slightly larger than the residual at 15 km. In Figure 9 we also show a breakdown of the total residual into contributions from separate inversions on the real and imaginary parts. The major characteristics of the total residual are determined by the residuals from the real part. The imaginary part shows far less sensitivity to the focal depth than the real part.

We plot the medium responses as a function of frequency for focal depths of 10, 15, 80, and 100 km in Figure 10. On the basis of the results in Figure 9 and Table 4 the main ingredient of the real part is \((\sqrt{\nu/\omega})G_1\), which has similar behavior across frequency for both shallow and deep foci. Consequently, if the response at 12 km matches the data well, we expect that it must also match well at 80-100 km.

It is surprising that the residuals from the imaginary part did not show any preference between shallow and deep foci. On the basis of the behavior of the response, \((\sqrt{\nu/\omega})G_3\), for shallow focus the amplitude of the imaginary part is about a factor of 10 greater at high frequency than at low frequency. For deep focus the amplitude of the response is at least a factor of 2 smaller at high frequency than at low frequency. The curve fits on the imaginary part for shallow and deep solutions are shown in Figure 11. The calculated amplitudes for 12 km match the observations better than the amplitudes for 80 km at the short periods (30 and 26 s). On the other hand, the amplitudes for 80 km match the observations better at long periods (60, 50 and 40 s). These characteristics of the fit (or lack of fit) plus the fact that the robust inversion placed more weight on the long-period observations explain why there is little difference between the size of the residuals for the shallow and deep focus solutions.

Despite the ambiguity in the residuals, there is strong evidence supporting a focus of 12 km from comparisons of the observed and calculated radiation patterns in Figure 12 and from re-
Fig. 9. Residual curve obtained from repeated application of the linear inversion. The residual plotted here is a weighted sum of squares computed by the robust method described by Patton [1978]. The horizontal dashed line refers to the total sum of squares (weighted) of the data on real part ($\mathcal{R}$) and on the imaginary part ($\mathcal{I}$).

examination of the amplitudes on the imaginary part. The greatest differences between the calculation patterns in Figure 12 are visible at short periods. Comparison with the observed amplitudes rejects the solutions for 90 and 100 km, but perhaps not 80 km. Table 5 gives normalized rms amplitudes of the imaginary part as a function of frequency for the solutions at 12 and 80 km. Calculation of the rms amplitude data on the imaginary part avoids the troublesome details of the phase data and allows discrimination only on the basis of the frequency dependence of the amplitudes. The observed rms amplitude increases about a factor of 2 from 40 to 26 s, which is consistent with a shallow focus. The deep focus predicts a factor of 2 decrease in the range of 40–26 s.

More evidence supporting the shallow focal depth comes from auxiliary data. Long-period P wave first motions (observed by this author) were plotted on a stereographic net for comparison with the fault planes and expected motions based on the solutions at 12 and 80 km. As seen in Figure 13, the solutions have principal stress directions rotated about 90° in such a way that the shallow focus has maximum compressive stress aligned nearly N–S and the deep focus has E–W alignment. Very different first-motion patterns are expected for these solutions. Clearly, the observed pattern of P wave first motions is incompatible with the pattern of the deep focus solution.

Results for all events: Depths and seismic moment tensors

The residuals as a function of trial depth for four of the eight remaining events are shown in Figure 14. All of the residual curves gave a minimum in the upper 15 km, which was interpreted as the effect of focal depth with the exception of event 6. The residual curve of event 6 has an absolute minimum at depths greater than 100 km. The local minimum at shallow focus for this event is caused by similar effects that gave a local minimum at great depths for event 9. In regard to the shape of the residual curves in the vicinity of the minimum, we note the following generalization: shallow events with broad minima, such as 1, 2, and 9, have strike slip mechanisms, whereas dip slip events such as 3 and 7 have sharp minima. Weidner [1972] found this to be true of the minima in $\mathcal{G}$, the residual from the analysis of differential phase. The reason that shallow focus dip slip events have stronger depth signature than strike slip may be understood by comparing medium responses $(\mathcal{V} \mathcal{V} \mathcal{V} \mathcal{V})_1$ and $(\mathcal{V} \mathcal{V} \mathcal{V} \mathcal{V})_2$ in Figure 3. The latter response, which is excited by the moment element $M_{zz}$, changes character more rapidly at shallow foci (0–15 km) than $G_1$ does. This means that in the frequency range (0.02–0.04 Hz) the Rayleigh wave complex spectrum is very sensitive to the depth of focus near the surface when it is made up of large component of $M_{zz}$, as in the case of dip slip mechanisms.

The seismic moment tensors of all nine events computed at the minimum of their respective residual curves are given in Table 6. In order to obtain a feeling for results of the linear
inversion we calculated the amplitude and phase radiation patterns for a few depths in the vicinity of the minimum residual and plotted them with the observed patterns. The plots are shown in Figures 15-19 for events 2, 4, 6, 7, and 8. We found that the solution at the minimum did not always give the best fit to the lnA and θ data. This was best seen on the plots of lnA for events 1, 2, 5, 6, and 9. Events 1 and 5 gave slightly better fits at trial depths 2.5 km from their minima. This small difference is probably an indication of the uncertainty in the estimate of focal depth due to random errors in the observations. The possibility of a bias in the estimate of the focal depth of event 9 due to epicentral location error is examined by Patton [1978]. Considerations of events 2 and 6 are given at further length below.

The following are remarks about specific events.

Events 2 and 4. Both events were too small to obtain P wave fault plane solutions. As seen in Figures 15 and 16, and Table 6, the mechanisms are clearly determined to be strike slip and dip slip, respectively, by the linear inversion and also by the trial and error logarithmic fit. In the case of event 2 the focal depths obtained by these two methods are 8 km different. Although an epicentral location error may be the cause, the combination of a strike slip mechanism and weaker signal strength of this small event adds greater uncertainty to the estimate of focal depth due to random errors. The results of event 4 clearly show that adequate sampling in
azimuth is needed to determine confidently all fault plane parameters. Here the slip vector favored by both the linear inversion and logarithmic fit has a large component of dip slip motion; however, owing to poor azimuthal sampling, a sizeable strike slip component is also permissible.

Events 7 and 8. These events were separated by 1.5 hours and less than 10 km in time and space. Yet their source mechanisms are very different, as shown by the results in Figures 18 and 19. The focal depths of both are 7.5 km. The linear inversion of event 7 gives primarily dip slip motion on a thrust fault. For event 8 the linear inversion result is left lateral strike slip on a very shallow dipping fault plane (or dip slip on a vertical plane). The direction of the principal axis in both mechanisms is aligned N-S, the axis of event 8 showing 30° greater dip from the horizontal plane than that of event 7.

Interestingly, both events 7 and 8 have large intermediate eigenvalues, as seen in the results of the inversions on Table 6. Patton and Aki [1979] demonstrated that multiplicative errors in the data can lead to large, significant departures from the double-couple force system. We checked the possibility that errors in our data were responsible for large intermediate eigenvalues by running logarithmic fits on the data sets of events 7 and 8. The results are shown with the results of the linear inversion on the amplitude and phase plots of events 7 and 8. The two results for event 8 are very similar. There is some indication that the frequency dependence of the amplitudes is not as well matched by the logarithmic fit as it is by the linear inversion. In the case of event 7 the logarithmic fit shows largest discrepancies with the linear fit at high frequencies in the azimuths northeast and southwest of the source, where station coverage is minimal. The linear fit shows some indication
Table 5. The Rms Amplitudes of Real and Imaginary Parts of Event 9, Observed and Theoretical

<table>
<thead>
<tr>
<th>Period, s</th>
<th>Real Part</th>
<th>Imaginary Part</th>
<th>Period, s</th>
<th>Imaginary Part</th>
<th>10 km</th>
<th>80 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.021</td>
<td>0.012</td>
<td>60</td>
<td>0.4</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>50</td>
<td>0.031</td>
<td>0.010</td>
<td>50</td>
<td>0.3</td>
<td>0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>40</td>
<td>0.033</td>
<td>0.018</td>
<td>40</td>
<td>0.6</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>34</td>
<td>0.032</td>
<td>0.025</td>
<td>34</td>
<td>0.8</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>30</td>
<td>0.034</td>
<td>0.024</td>
<td>30</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>26</td>
<td>0.039</td>
<td>0.030</td>
<td>26</td>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

of a slightly better fit to the focal phase at high frequencies. Although it is tempting to conclude from the recovery of a large, apparently significant intermediate eigenvalue that there is a real departure from the double couple mechanisms, as others [Randall and Knopoff, 1970; Dziewonski and Gilbert, 1974] have proposed, we find it difficult to establish this convincingly in the light of errors in our data and the closeness to the results of the double-couple models.

Event 6. The residual curve indicated that its focal depth is greater than 100 km. This is in agreement with the ISC reported source depth of 111 km. On the amplitude plots in Figure 17 we show results of the linear inversion at trial depths of 60, 80, and 100 km. The frequency content of the observed amplitudes are matched well by the 100-km focus model except the highest frequency, where the calculated is too low. Deeper models will not improve this because the amplitude of the normal modes at these depths in the earth model monotonically approach zero. Unless the high observed amplitudes can be accounted for by other means, we would have to conclude that this is a failing of the Pamir earth model assumed in the calculation of the response functions for Central Asia. Although no attempt was made to improve the fit by changing the Pamir model, it should be possible to do so when there is more knowledge about the earth structure of Central Asia.

There are several general comments to make about the results in Table 6. Comparison of the revised source parameters of events 1 and 3 with the parameters obtained from the earthquake pair method in Table 3 is quite close. The depths show little or no change, and the seismic moments are different by less than 25%. The estimates of the moment element $M_{xz}$ show the largest change among all elements for both events. These are signs that the iterative process converged rapidly upon stable estimates of the path parameter for most paths. The fact that $M_{xz}$ radiation pattern is two lobe

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**Fig. 13.** P wave first motions of event 9 plotted on a stereographic net; see caption of Figure 4 for definition of symbols. Shown with these observations are the fault planes and principal stress axes based on results of the inversion at shallow and deep foci.
oriented east-west indicates that convergence was slowest for the tectonic east-west paths, suggesting a correlation between speed of convergence and the complexity of the path. In regard to the principal axes of all events we call attention to the fact that the orientation of the P axis, $\varepsilon_3$, is very constant. The largest deviation from N-S alignment is $43^\circ$ in the case of deep event 6. Among the shallow events, the event 9 P axis shows the largest deviation at $24^\circ$W. The maximum deviation of the P axis from horizontal is for events 6 and 8 at $43^\circ$. Among shallow events it is fair to conclude that P axes are typically oriented N-S and horizontal. Further interpretation of the focal mechanisms in light of the faulting and tectonics of this area is discussed by Jackson et al. [1979].

6. Propagation Parameters

For purposes of examining the path parameters the transfer functions $H_{ik}e^{i\phi_{ik}}$ were converted into apparent phase velocities and attenuation coefficients. Apparent phase velocities were calculated by the following equation:

$$c_{ik}(\omega_k) = \frac{\omega_k}{K_{1k}} = \frac{\omega_k \bar{X}_i}{\phi_{ik}}$$  \hspace{1cm} (16)

where $\omega_k$ is frequency, $K_{1k}$ is wave number, $\bar{X}_i$ is the distance from the reference point to the ith receiver, and $\phi_{ik}$ is an estimate of the propagation phase (total number of cycles including the order number). The apparent attenuation coefficient $\eta_i(\omega_k)$, which is proportional to $Q^{-1}$, is computed as follows:

$$\eta_i(\omega_k) = \frac{-\ln(\tilde{H}_{ik})}{\bar{X}_i}$$  \hspace{1cm} (17)

where $\tilde{H}_{ik}(\omega_k)$ is an estimate of the amplitude transfer function. The results of two calculations, one obtained from an estimate of $H_{ik}$ using (14) (maximum likelihood estimate, MLE) and the other using (13) (LAV, i.e., log average), are compared in Figure 20 for a representative set of stations. We give error bars, representing one standard deviation in the calculation of LAV of $H_{ik}$, at three selected frequencies (1/50, 1/34, and 1/26 Hz). The estimates of $\eta$ based on the MLE calculation are shown by open circles joined with straight lines in order to enhance the behavior of this estimate from one frequency to the next. For reasons given below, the behavior of the MLE, such as smoothness and variations from the LAV estimate, is an indication of the statistical properties of noises on the seismogram.
Table 6. Focal Depth and Seismic Moment Tensor Giving Smallest Residual

<table>
<thead>
<tr>
<th>Event</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Depth, km</td>
<td>7.5</td>
<td>15</td>
<td>5</td>
<td>7.5</td>
<td>10</td>
<td>100</td>
<td>7.5</td>
<td>7.5</td>
<td>15</td>
</tr>
<tr>
<td>Seismic moment, dyn cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{xx}$</td>
<td>$x10^{24}$</td>
<td>$-0.5\pm0.1$</td>
<td>$-0.21\pm0.07$</td>
<td>$1.5\pm0.1$</td>
<td>$4.1\pm0.6$</td>
<td>$1.2\pm0.1$</td>
<td>$-1.1\pm0.3$</td>
<td>$1.9\pm0.2$</td>
<td>$-1.1\pm0.3$</td>
</tr>
<tr>
<td>$M_{yy}$</td>
<td>$x10^{24}$</td>
<td>$-1.75\pm0.10$</td>
<td>$-2.3\pm0.1$</td>
<td>$-4.3\pm0.4$</td>
<td>$-1.2\pm0.1$</td>
<td>$1.3\pm0.6$</td>
<td>$-4.2\pm0.2$</td>
<td>$-5.0\pm0.4$</td>
<td>$-5.4\pm0.3$</td>
</tr>
<tr>
<td>$M_{xy}$</td>
<td>$x10^{24}$</td>
<td>$0.33\pm0.06$</td>
<td>$-0.1\pm0.1$</td>
<td>$0.4\pm0.5$</td>
<td>$0.3\pm0.1$</td>
<td>$1.8\pm0.6$</td>
<td>$0.3\pm0.1$</td>
<td>$0.4\pm0.2$</td>
<td>$3.5\pm0.2$</td>
</tr>
<tr>
<td>$M_{xz}$</td>
<td>$x10^{24}$</td>
<td>$0.6\pm0.2$</td>
<td>$-0.29\pm0.04$</td>
<td>$0.8\pm0.3$</td>
<td>$0.5\pm0.3$</td>
<td>$0.0\pm0.1$</td>
<td>$-0.1\pm0.2$</td>
<td>$0.2\pm0.2$</td>
<td>$2.0\pm0.4$</td>
</tr>
<tr>
<td>$M_{yz}$</td>
<td>$x10^{24}$</td>
<td>$0.2\pm0.2$</td>
<td>$-0.30\pm0.05$</td>
<td>$2.8\pm0.4$</td>
<td>$2.0\pm0.4$</td>
<td>$1.1\pm0.1$</td>
<td>$1.3\pm0.3$</td>
<td>$1.3\pm0.3$</td>
<td>$6.0\pm0.5$</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td></td>
<td>$2.8\pm0.1$</td>
<td>$1.1\pm0.06$</td>
<td>$3.2\pm0.3$</td>
<td>$4.6\pm0.6$</td>
<td>$1.6\pm0.1$</td>
<td>$2.8\pm0.6$</td>
<td>$2.3\pm0.2$</td>
<td>$5.7\pm0.5$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td></td>
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<td>$-0.2\pm0.07$</td>
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<tr>
<td>$\lambda_3$</td>
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<td>$-3.5\pm0.4$</td>
<td>$-4.6\pm0.5$</td>
<td>$-1.7\pm0.1$</td>
<td>$-2.2\pm0.4$</td>
<td>$-3.4\pm0.2$</td>
<td>$-7.7\pm0.5$</td>
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<td>$e_1$</td>
<td></td>
<td>$-102^\circ$</td>
<td>$78^\circ$</td>
<td>$-154^\circ$</td>
<td>$-151^\circ$</td>
<td>$-171^\circ$</td>
<td>$-148^\circ$</td>
<td>$-135^\circ$</td>
<td>$-132^\circ$</td>
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<td>$e_2$</td>
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<td>$278^\circ$</td>
<td>$-45^\circ$</td>
<td>$103^\circ$</td>
<td>$88^\circ$</td>
<td>$83^\circ$</td>
<td>$-252^\circ$</td>
<td>$90^\circ$</td>
<td>$119^\circ$</td>
</tr>
<tr>
<td>$e_3$</td>
<td></td>
<td>$75^\circ$</td>
<td>$65^\circ$</td>
<td>$8^\circ$</td>
<td>$8^\circ$</td>
<td>$6^\circ$</td>
<td>$43^\circ$</td>
<td>$14^\circ$</td>
<td>$21^\circ$</td>
</tr>
</tbody>
</table>

Patton: Reference Point Method for Surface Waves
Fig. 15. Results of the linear inversion for event 2 at focal depths of 10 km (solid lines) and 15 km (dashed lines); results of logarithmic fit (dot-dashed lines) at 7 km focus; (a) amplitude patterns; (b) focal phase patterns.
Fig. 16. Results of the linear inversion for event 4 at focal depths of 5 km (dot-dashed lines) and 10 km (dashed lines); results of logarithmic fit (solid lines) at 6 km focus; (a) amplitude patterns; (b) focal phase patterns.
Fig. 17. Results of the linear inversion for event 6 at focal depths of 60 km (dot-dashed lines), 80 km (dashed lines), and 100 km (solid lines); (a) amplitude patterns; (b) focal phase patterns.
Fig. 18. Results of the linear inversion for event 7 at 7.5 km focus with a 2 s origin time correction (solid lines) and without (dashed lines). Logarithmic fit (source parameters: $h = 7.5$ km, $s = -36^\circ$, $d = 34^\circ$, $\theta_F = 240^\circ$, and $M_0 = 4.4 \times 10^{24}$) is shown by long-dashed lines; (a) amplitude patterns; (b) focal phase patterns.
Fig. 19. Results of the linear inversion for event 8 at focal depths of 5 km (dot-dashed lines), 7.5 km (solid lines), and 12.5 km (dashed lines). Logarithmic fit (source parameters: \( h = 10.5 \) km, \( s = 36^\circ \), \( d = 50^\circ \), \( \phi_r = 210^\circ \) and \( M_0 = 6.1 \times 10^{24} \)) is shown by long-dashed lines.
When interference occurs on the seismogram, phase errors are introduced into the signal spectrum. These errors can be very damaging in the MLE calculation. This is because large-amplitude signals, which are weighted heavily in the MLE, suffer signal-generated noises more than small signals do on any common path. Large phase errors, of course, do not preclude the possibility of ambient noise, especially in small signals. However, in that circumstance the weighting in the MLE is appropriate to minimize the effects of these errors. When phase errors are small, there is still a possibility of a bias in the MLE due to large magnification errors. In this case the MLE is expected to give a low estimate of the attenuation coefficient because the observed amplitude distribution is skewed to higher amplitude by the magnification errors.

In Figure 20 the MLE's computed from GDH and UME are mildly variable at high frequencies. More erratic behavior is seen on plots of PTO. TRI, HKC, and NAI. In general, suspicious behavior of the MLE correlates well with the occurrence of large phase errors denoted on these plots. Our interpretation is that the noises over the frequency range where this occurs are predominantly signal generated in nature, such as multipathing interference. The station MSH shows a large spread of amplitudes, as the standard error on  indicates, and yet the phase error is small. Compared with the log average result, the MLE method underestimates , as expected. Here our interpretation is that magnification errors, perhaps due to focusing of seismic rays along azimuths to MSH, are causing the amplitude distribution to be skewed.

Azimuthal variations in $C_{ik}$ and $H_{ik}$

There are some interesting correlations between the measurements of propagation parameters and the various geologic and geographic provinces in Eurasia. The correlations are visible on three sets of plots in Figure 21. Each plot shows the measurements of attenuation coefficients and phase velocities as a function of azimuth for periods 50, 34 and 26 s. Error bars on the attenuation coefficients are computed from the LAV method. A scale showing 1% variation is given for reference on the plot of phase velocities. On the plot of attenuation coefficients the horizontal line corresponds to an average value of attenuation obtained in section 4 from the slope of the line that fits the decay of the log amplitudes plotted against epicentral distance. The horizontal line on the plot of phase velocities corresponds to the value of phase velocity for the Gutenberg continental earth model.

Perhaps the most obvious feature in Figure 21 is that the azimuthal variations of apparent attenuation and phase velocity increase with frequency. For example, the total percentage variation of phase velocity is 9%, 13%, and 15% at 50-, 34-, and 26-s periods, respectively. Paths with consistently the highest velocities in Eurasia are those crossing the Russian Platform and Norwegian Sea. The lowest velocities are measured on paths crossing the Tibetan Plateau and the Hindu Kush. The highest attenuation in Eurasia occurs at short periods on paths through the Middle East, Iraq, and Iran. Stations in India showed high attenuation at short and long periods. The lowest apparent attenuation may be seen at short periods for paths through the Alpine forelands and fold belt systems extending from central Europe through the Adriatic and including the Alpide fold.
belts, the Crimes, and the Caucasus. Anomalously low apparent attenuation is measured at long periods for paths crossing the Tibetan Plateau, Indonesia, and southern China.

Azimuthally, from left to right across the plot, the phase velocities at long period drop steadily from the paths on the Russian Platform, the Alpine forelands, and central European paths, and the southern European paths through Alpine-Alpide fold belt systems. This trend appears to bottom out on paths crossing the Middle East, Iraq, and Iran. Indian Shield paths have the highest velocities in southern Asia. For paths east of the reference point, the variation of velocities at long period as well as at short period is controlled by the percentage of the path in the Tibetan Plateau. Paths across northern China show the highest velocities of all eastern paths.

At short period, all but two paths show apparent velocities below the average continental velocities represented by the Gutenberg earth model. The velocities over paths interior to the continent on the Russian Platform around 60° of azimuth show little variation. This includes many southern paths through the tectonic Alpine-Alpide fold belts. A rapid transition is apparent at 285° azimuth near the stations IST and ATU. To the right of this transition, paths through the Middle East Iraq and Iran show little variation in velocities again.

Apparent attenuation at long period shows little variation about the mean value, especially for western paths. Eastern paths show variation in inverse relationship to the phase velocities: for Tibetan paths we see low velocities and high Q, and for northern China we see high velocities and low Q. South of the reference point, Indian stations also show high phase velocities and low Q. At long period only three western paths deviate significantly from the mean attenuation. One path crosses the Norwegian Sea to station KTG. The other two paths cross the Russian Platform to adjacent stations in Scandinavia, NUR and KON.

For periods 34 and 26 s we have connected the attenuation coefficients for paths that cross the Norwegian Sea separately from the other measurements. Except for high apparent attenuation at two stations in Scandinavia, KEV and UME, all paths interior to the continent crossing the Russian Platform, Alpine forelands, and fold belt systems show lower attenuation than the Norwegian Sea paths. Nevertheless, attenuation of the continental paths south of the reference point is even higher than that observed at stations across the Norwegian Sea. Most southern paths regardless of the type of landform show much higher attenuation than the mean value at high frequencies. There is evidence of extremely rapid variations of apparent attenuation on paths to stations in Burma and Indonesia. The measurements of attenuation coefficients on three paths through northern China are remarkably constant over the frequency range.

Some features of the propagation data are difficult to reconcile. For most northwestern paths on the continent we see high velocities and high Q, which is consistent with expected correlations between these data. However, on southern and eastern paths there are many observations showing the opposite correlation, i.e., low velocity and high Q or high velocity and low Q. A future paper concerning the lateral variation of phase velocity and Q on the Eurasian continent will address these features in further detail.
7. Conclusions
This paper has described a reference point equalization method for the determination of source and propagation effects of surface waves. The method consists of two essential steps: initialization and iteration. Initialization obtains the first reference events needed to compute initial estimates of phase velocity and attenuation coefficients along individual paths between the reference point and the receivers. Iteration simultaneously refines the path parameters and determines the source parameters of new earthquakes in the vicinity of the reference point. The application of this method to Central Asian earthquakes has demonstrated that it works well for studying the source mechanism of remote earthquakes and the nature of wave propagation between the reference point and surrounding receivers. Among the main conclusions drawn from this study are the following:

1. The results of the least squares moment tensor inversion agreed well with the source parameters determined from the logarithmic fit when precautions were taken to remove bad data points. A modification of the straight least squares inversion was used to make the inversion insensitive to the presence of bad data points. This robust method, involving weighted least squares, is not as fast as the straight least squares approach; however, it still represents a significant saving in computer time over the trial and error fitting.

2. The residuals obtained from the repeated application of the moment tensor inversion over trial focal depths showed two minima; one minimum occurred at depths less than 20 km, and the other at depths greater than 70 or 80 km. The values of the residuals at these minima were often close enough to case doubt on the determination of focal depth. The ambiguity is due to combination of factors, one of which is the similarity of the response, $(V/U_0)\times I_0$, at shallow and deep focus over the frequency range of this study. We were able to resolve the ambiguity by examining the behavior of the rms amplitudes on the imaginary part and by comparing the geometry of the moment tensor obtained for shallow and deep focus inversions with the observed P wave polarities.

3. The linear moment tensor inversion generally gives three-couple force systems having a significant nonzero intermediate component. This may be expected on the basis of the results obtained from numerical modeling of the effects of noise contamination on the linear inversion [Patton and Aki, 1979]. Comparisons of source effects of double-couple source models and the three-couple models are very close, and in the light of errors in our data it cannot be established convincingly that the results of the linear inversion are caused by actual departures of the source from the double-couple model.

4. The focal depths of eight earthquakes in the Pamirs, Central Asia, were determined to be 5-15 km. The principal compressive stress axes of these earthquakes were consistently found to be oriented north-south and close to horizontal. This feature of the stress field in the Pamir is in agreement with the interpretation that present tectonic activity in Central Asia is due to the north-south convergence of the Eurasian and Indian plates [Dewey and Bird, 1970; Molnar et al., 1973; Molnar and Tapponnier, 1975].

5. Apparent phase velocities show strong azimuthal variation that is well correlated with geologic and geographic features on the Eurasian continent. The lowest apparent velocities in Eurasia are on paths over Tibet and the Hindu Kush. The highest apparent velocities are measured over paths on the northern platforms. The differences in these velocities is 9% at 50 s, 13% at 34 s, and 15% at 26 s. Apparent attenuation at 50-s period shows relatively minor azimuthal variation compared to the attenuation at the short period (26 s). At the short period, attenuation over continental paths in southern Asia is higher than it is on paths crossing the Mid-Atlantic Ridge in the Norwegian Sea. Northern paths confined mainly to the platforms have lower attenuation at short period than either Norwegian Sea paths or southern paths.

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